



NARODOWY
BANK POLSKI

NBP Working Paper No. 353

Monetary policy in a two-country model with behavioral expectations

Michał Brzoza-Brzezina, Paweł Galiński, Krzysztof Makarski



NBP Working Paper No. 353

Monetary policy in a two-country model with behavioral expectations

Michał Brzoza-Brzezina, Paweł Galiński, Krzysztof Makarski

Michał Brzoza-Brzezina – Narodowy Bank Polski; michal.brzoza-brzezina@nbp.pl
Paweł Galiński – Narodowy Bank Polski; pawel.galinski@nbp.pl
Krzysztof Makarski – Narodowy Bank Polski; krzysztof.makarski@nbp.pl

The views expressed herein are those of the authors and not necessarily those of Narodowy Bank Polski. We appreciate the comments received from Zeno Enders, Michał Gradzewicz, Marcin Kolasa, an anonymous referee and from the participants of the seminar at Narodowy Bank Polski, the Summer Workshop on Macroeconomics and Finance and the NBP-BoL-CEBRA conference “New Dimensions of Monetary Policy”.

Published by:
Narodowy Bank Polski
Education & Publishing Department
ul. Świętokrzyska 11/21
00-919 Warszawa, Poland
www.nbp.pl

ISSN 2084-624X

© Copyright Narodowy Bank Polski 2022

Contents

Abstract	4
1 Introduction	5
2 Literature review	7
3 Model	10
3.1 Households	10
3.2 Linearization	11
3.3 Producers	12
3.3.1 Consumption good producers	12
3.3.2 Intermediate goods producers	13
3.4 Government and the central bank	14
3.5 Closing the model	15
3.5.1 GDP and Balance of Payments	15
3.5.2 Market clearing	15
4 Calibration and estimation	17
4.1 Calibrated parameters	17
4.2 Data and prior assumptions	17
4.3 Estimation results	18
5 Consequences of behavioral assumptions for monetary policy	19
5.1 The working of monetary policy	19
5.2 The monetary policy trade-off	20
5.3 Determinacy of monetary policy	21
6 Behavioral agents and open-economy puzzles	22
6.1 Excess returns (UIP puzzle)	22
6.2 International monetary policy independence	23
7 Robustness	24
8 Conclusions	25
References	26
Tables and figures	30
Appendix A	36
A.1 Derivation of the Euler and UIP equations	36
A.2 Derivation of the Phillips curve	37
A.3 Model equations	38
A.3.1 Households	38
A.3.2 Producers	38
A.3.3 Government	41
A.3.4 Central Bank	41
A.3.5 Closing the model	41

Abstract

We study the working of monetary policy in an estimated two-country model with behavioral expectations (BE). We first show that the data favors this setting compared with the standard rational expectations assumption. Then we document several findings related to monetary policy in the open-economy framework. First, under BE the Taylor principle depends on the size of the economy - determinacy regions are larger for the small country. Second, both in the small and large economies, monetary policy is less powerful when agents are behavioral. Third, the sacrifice ratio faced by the central bank increases with agents becoming more behavioral (more in the small country). Fourth, BE help to partly solve the puzzles of excess foreign currency returns (UIP puzzle) and of international monetary independence.

JEL: E30, E43, E52, E70

Keywords: behavioral agents, monetary policy, open-economy model

1 Introduction

In spite of the mounting evidence that people do not act rationally, most monetary economists (including some of this paper’s authors) have been constructing models with the central assumption of agents forming expectations in a rational fashion. To make sure, this was not necessarily wrong. Building models is always an act of striking a balance between simplicity and adequacy for answering the question at hand. Simplifying assumptions are warranted (even welcome) as long as the findings are robust to them. In the context of monetary policy research the problem with rational expectations (RE) is that, in spite of extensively using this assumption, we still know little how robust our conclusions are in this context.

One of the main, well known problems, with dropping the RE assumption is, that while one can be rational in one way only, one can be non-rational in a huge number of fashions. As a consequence modeling away from RE includes a great deal of discretion and possible confusion. Nevertheless, recently a number of studies emerged that try to explore the consequences of non-rationality in various areas of economics, including monetary issues. Section 2 reviews the key recent directions of research, here we concentrate on the path that shaped our paper. This path, introduced by Gabaix (2020) assumes an informational (cognitive) friction that makes people myopic with respect to future developments. Gabaix (2020), as discussed below, showed in a closed economy model that such deviation from the RE assumption helps solving several counterfactual findings from the business cycle literature, including overly strong effects of forward guidance (Del Negro et al., 2012), restrictive determinacy conditions for monetary policy rules as well as overly low fiscal multipliers. He calls his agents “behavioral”, and we follow this terminology.¹

We find the framework promising enough to test whether its implications are as path-breaking in the open-economy context. To this end we construct a two-country model, which, apart from the behavioral features is a standard New Keynesian framework with sticky prices that has been frequently used for monetary policy analysis. Leaving a detailed literature review to the next Section it is worth mentioning, that the open-economy aspect of behavioral expectations (BE) has so far been almost untouched by the profession. The closest paper to ours is Kolasa et al. (2022), which however works with a calibrated, small open economy model, leaving questions that only a multi-economy or an estimated framework can handle unanswered. In asking and answering such questions, related in particular to the working of monetary policy and the solution of open economy macro puzzles we see our main contribution to the literature.

¹This may be slightly confusing, as the term “behavioral expectations” has also been used in the literature with an alternative meaning. Hommes et al. (2019) call “behavioral” agents who follow a heuristic switching model when forming expectations.

We estimate the model using data for Poland (small economy, SE) and the euro area (large economy, LE). Bayesian estimation and comparison with a similar model with RE assumed shows that the BE assumption is strongly favored by the data. Then we conduct a number of experiments, which show that:

- under BE determinacy regions for monetary policy are not only larger than under RE, but also depend on the size (openness) of the economy. In general more passive monetary policy is allowed if the economy is small. This is a consequence of the real exchange rate acting as an additional stabilizing device under BE,
- under BE reactions to monetary policy shocks are weaker in the short run and more persistent in the longer term than when agents act rationally. This is true both for small and large economies and mainly the consequence of agents not properly anticipating the future consequences of contemporaneous monetary policy shocks,
- with expectations becoming more behavioral, the monetary policy trade-off worsens: in both the small and large economies stabilizing inflation comes at an increasing cost of destabilizing output (i.e. the sacrifice ratio increases), but the effect is much more pronounced in the SE. This results from the reduced role of the expectations channel of monetary policy,
- BE allows to solve one of the most important puzzles in open-economy macroeconomics (the UIP puzzle): after an unexpected increase of the domestic interest rate excess returns on the domestic currency can be achieved for several periods (which stands in contrast to the conclusion stemming from RE that excess returns are unforecastable),
- the BE model takes a step towards aligning theory and practice of international monetary policy independence. Under RE in response to a contractionary monetary policy shock in the LE, policymakers in the SE lower interest rates. This is inconsistent with empirical evidence. In the BE model monetary policy in the SE is tightened, although the effect is not strong enough to solve the puzzle completely.

All in all we show that the behavioral framework is strongly preferred by the data in the open-economy context and that it brings a number of implications and improvements that both policymakers and modelers of open economies should find important and interesting.

The rest of the paper is structured as follows. Section two presents a review of the relevant literature. Section three describes the model and section four its calibration/estimation. Sections five and six discuss the implications of agents being behavioral for monetary policy in the small and large economies as well as for open-economy macro puzzles. Section seven discusses the robustness and Section eight concludes.

2 Literature review

The rational expectations hypothesis, proposed by Muth (1961), significantly impacted the macroeconomic literature. It assumes that economic agents use all available data to construct statistically optimal and consistent expectations about the future. Muth argued that even if people are not perfect forecasters, they generally do not waste information and that all behavioral biases have only a short-term and non-significant impact on the decision-making process. Lucas (1972) and Kydland and Prescott (1982) helped to implement this approach into mainstream macroeconomic modeling and claimed that even if rational expectations are an abstract concept, they allow economists to discipline the discussion about expectations and modeling. As a consequence, the rational expectations assumption shaped economic modeling for the decades to come.

However, a growing behavioral economics literature, initiated by Daniel Kahneman and Amos Tversky, argues that people are prone to systematic cognitive biases that significantly affect economic agents' decision-making process. Kahneman and Tversky (1972) showed that while making judgments, people systematically deviate from the Bayesian probability distribution, a behavior they called heuristics. Heuristics are simplified processes of reasoning and decision-making based on "rule of thumb" behavior used by all people on a daily basis. This phenomenon is generally beneficial as it allows us to make quick decisions instead of conducting an in-depth analysis of each situation. However, heuristics lead also to cognitive biases such as the anchoring bias, confirmation bias, conjunction fallacy, base rate neglect, or overreaction.

After the 2007-08 financial crisis, it became clear that not only do those biases have a short-term impact on people's decisions but also can exert serious macroeconomic pressures. Non-rational forecasts are seen as an important cause of the growth of financial bubbles (Greenwood and Hanson, 2013; Baron and Xiong, 2017; Fahlenbrach et al., 2017) whose burst can lead to deep recessions (López-Salido et al., 2017; Jordà et al., 2015; Greenwood et al., 2020). The over-reaction to information plays an important role in this phenomenon because, on the one hand, it sustains bubbles before the crisis and, on the other, leads to a stronger economic collapse than in RE models (Gennaioli and Shleifer, 2018). The non-rationality of expectations also occurs at the level of enterprises, which construct too optimistic expectations in times of good economic conditions (negating the risk) and too pessimistic expectations when the economic situation is poor (assuming too much risk), which can effectively lead to more severe recessions and longer economic recovery (Gennaioli et al., 2015; Bordalo et al., 2021; Coibion and Gorodnichenko, 2015).

Against this background, several approaches have been taken to accommodate non-rational expectations in macroeconomic modeling. A relatively early approach, adaptive learning, assumes agents acting as econometricians that perform ordinary least squares re-

gressions based on available data (Evans and Honkapohja, 1999; Eusepi and Preston, 2011; Pintus and Suda, 2019). Learning models match the data better than RE models but are not based on psychological discoveries. Recently more attention is being paid to approaches that refer to cognitive limitations of agents. An early example of such an approach is the rational inattention theory proposed by Maćkowiak and Wiederholt (2015), where decision-makers are rational; however, they have limited attention to information that they have to allocate between different decisions. Using rational inattention in DSGE models provides several predictions, including the result where households pay less attention to an interest rate which impacts the way prices respond to different shocks.

More recent examples of accounting for cognitive limitations include theories of k-level thinking, diagnostic expectations and cognitive discounting. K-level thinking, derives from the artificial intelligence field, especially from advanced chess programs (Woodford, 2019). Such programs, after each move, create an advanced decision tree that includes future moves. However, obtaining a tree that includes all possible future moves is impossible, therefore chess engines are designed to forecast a finite number of moves. The same reasoning is adapted to model non-rationality of economic agents that create rational forecasts, but only for a finite k-number of periods. Implementing k-level thinking to economic modeling leads to modification of monetary policy effectiveness, as k-level agents are less sensitive to forward guidance (García-Schmidt and Woodford, 2019; Woodford, 2019). However, a possible problem related to this approach is that it primarily relates to artificial intelligence rather than the achievements in the psychology field. One can claim that such behavior (decision trees) can be found in the humans' decision-making process; however, the connection between such approach and psychology is not well investigated.

In contrast, strongly nested in psychology is the diagnostic expectations theory (DE), based on the representativeness heuristics that leads people to pay too much attention to the most accessible information (Tversky and Kahneman, 1974). Therefore, DE assumes that while making decisions, economic agents attach too much weight to information from the nearest preceding periods, constructing expectations based on a nonobjective set of information. For this reason, diagnostic agents overreact to shocks, which contributes to stronger economic fluctuations (Bordalo et al., 2020). Models with DE can explain much more volatility than models with RE, especially in the case of boom-bust cycles analysis (L'Huillier et al., 2021). Therefore, DE is well-designed to model short-term macroeconomic fluctuations in models with financial markets. However, implementing them to the New Keynesian model does not fundamentally change its properties.

In this paper, we focus on the concept of behavioral agents proposed by Gabaix (2020), featuring economic agents who construct forecasts in a myopic fashion. Gabaix calls this phenomenon cognitive discounting and provides microfoundations based on noisy information processing. As compared to RE, agents shrink their simulations of the future towards the

steady-state of the economy (Gabaix and Laibson, 2017; Gabaix, 2019). Gabaix (2020) incorporates behavioral agents into the New Keynesian closed economy model to check if it can help explain some economic puzzles. Accordingly, in the NK model with myopic agents (i) fiscal policy is more powerful than in rational models, as agents fail to anticipate future tax increases after receiving government transfers, (ii) the Taylor principle is modified and allows the central bank to be passive in response to inflation shocks, (iii) it is possible to explain the stability of economies stuck at the zero lower bound (ZLB), (iv) the impact of forward guidance is reduced.

A literature that implements behavioral agents into macroeconomic models follows. For instance Bounader and Traficante (2022) show that under non-rational expectations monetary policy should act more aggressively to stabilize the economy. However most results come from closed economy frameworks, and therefore the question of the impact of cognitive discounting in open economy models remains open. So far open economy issues have been taken up by Kolasa et al. (2022), who use a calibrated, small open economy model with behavioral agents. They show that implementing behavioral agents allows for solving economic puzzles, including the forward premium (UIP) puzzle (Fama, 1984), the predictability reversal puzzle (Bacchetta and Van Wincoop, 2010), the Engel puzzle (2013) and the forward guidance exchange rate puzzle (Gali, 2020). Moreover, cognitive discounting lowers the efficacy of forward guidance. Our paper complements the existing literature by allowing for conclusions from an estimated two-country model with behavioral agents.

3 Model

We study the working of monetary policy in a two country model with imperfect rationality. To this end we employ the cognitive discounting approach of Gabaix (2020) (for exhaustive technical details see the aforementioned paper). The main idea of this approach is that agents perceive future deviations from the steady state imperfectly. Let us express the interest rate in period $t + k$ as $r_{t+k} = \bar{r} + \hat{r}_{t+k}$, where \bar{r} denotes the steady state value and \hat{r}_{t+k} its deviation from the steady state. Behavioral expectations (denoted by E^{BE}) of the future deviations from the steady state are shrunk by a factor m^k , relative to rational expectations, where $m \in [0, 1]$ captures the extent of cognitive discounting $E_t^{BE} r_{t+k} = E_t^{BE}(\bar{r} + \hat{r}_{t+k}) = E_t(\bar{r}) + m^k E_t(\hat{r}_{t+k})$.

Other than for cognitive discounting, our framework is a standard two country DSGE model with nominal rigidities. We call one of them domestic and the other foreign. Measure ω of agents reside in the former and $\omega^* = 1 - \omega$ in the latter one. Both economies are populated by households, producers of consumption goods for domestic and foreign markets, monetary authorities and governments. Prices are set as proposed by Calvo (1983). In this paper we employ the following notation convention: variables without an asterisk refer to the home country, while variables with an asterisk pertain to the foreign one. Since both countries have a symmetric structure, we describe the problems of agents in the home country only. Throughout the paper variables with bars denote the steady state values, variables with hats denote deviations from the steady state. We employ the convention that the variables denoted with small letters denote the real value of their nominal analogs denoted with capital letters. Selected derivations are presented in more detail in the Appendix.

3.1 Households

Households supply labor, consume goods and trade domestic and foreign bonds. They form expectations about future variables via cognitive discounting. The representative household's preferences are as follows

$$E_0^{BE} \left\{ \sum_{t=0}^{\infty} \varepsilon_{u,t} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \varepsilon_{n,t} \frac{n_t^{1+\gamma}}{1+\gamma} \right) \right\} \quad (1)$$

where β denotes the discount rate. c_t and n_t , denote consumption and labor supply, respectively. The inverse of the intertemporal elasticity of substitution in consumption is denoted by σ , while γ is the inverse of the Frisch elasticity of labor supply. Furthermore, there is an intertemporal preference shock $\varepsilon_{u,t}$ following an AR(1) process, with persistence ρ_u and standard deviation of innovations σ_u .

Each household has access to both domestic and foreign bond markets. Domestic bonds $B_{H,t}$ offer a risk free nominal interest rate R_t and foreign bonds² $B_{H,t}^*$ offer a risk free nominal rate $R_t^* \kappa_t$, where R_t^* denotes the interest rate abroad and κ_t a risk premium. The risk premium is a function of the aggregate domestic debt (as in Schmitt-Grohe and Uribe, 2003) and each household takes it as given

$$\kappa_t = \exp \left(-\chi \left(\frac{-S_t B_{H,t}^*}{P_t \tilde{y}_t} - d \right) \right) \varepsilon_{\kappa,t}, \quad (2)$$

where S_t , and \tilde{y}_t denote the nominal exchange rate and GDP, respectively. The constant d is calibrated so that there is no risk premium in the deterministic steady state. $\varepsilon_{\kappa,t}$ denotes a risk premium shock that follows an AR(1) process with persistence ρ_κ and standard deviation of innovations σ_κ . We also assume that households own all domestic firms in the economy, which pay them dividends with real value Γ_t .

All domestic households have the same budget constraint in each period

$$P_t c_t + B_{H,t} + S_t B_{H,t}^* + P_t T_t \leq W_t n_t + R_{t-1} B_{H,t-1} + S_t \kappa_{t-1} R_{t-1}^* B_{H,t-1}^* + P_t \Gamma_t \quad (3)$$

where W_t and P_t denote the nominal wage and the consumption good price, respectively. T_t denotes the real value of lump sum taxes levied by the government.

Households maximize preferences (1) subject to the budget constraint (3). In order to solve their problem it is convenient to define the real values of domestic and foreign bond holdings as $b_{H,t} \equiv B_{H,t}/P_t$ and $b_{H,t}^* \equiv B_{H,t}^*/P_t^*$. Inflation is denoted as $\pi_t = P_t/P_{t-1}$.

3.2 Linearization

While linearizing the model we introduce the following notation. To allow for the case of zero bond holdings in the steady state, domestic and foreign bonds are expressed as level deviations from the steady state, e.g. $\hat{b}_{H,t} = b_{H,t} - \bar{b}_H$. The remaining variables are expressed as percentage deviations from the steady state, i.e. $\hat{X}_t = (X_t - \bar{X})/\bar{X}$. The solution of the household problem³ gives rise to the following Euler equation expressed in linearized form

$$\begin{aligned} \hat{c}_t &= m E_t \hat{c}_{t+1} - \frac{1}{\sigma} (\hat{R}_t - m E_t \hat{\pi}_{t+1}) + \frac{1}{\sigma} (\hat{\varepsilon}_{u,t} - m E_t \hat{\varepsilon}_{u,t+1}) \\ &+ \frac{(1-\beta)}{\beta} (1-m) \frac{1}{(\bar{c} + \frac{1}{\mu} \frac{\sigma}{\gamma})} (\hat{b}_{H,t} + \hat{b}_{H,t}^* + \bar{b}_H^* \hat{\kappa}_t) \end{aligned} \quad (4)$$

This features two key modifications as compared to the RE version. First, as in Gabaix (2020), expectations of next-period variables are discounted in line with the myopic behavior

²We calibrate the model to match the data, so that the home country is a borrower in the neighborhood of the steady state and the foreign country is a lender.

³The details of the derivation are presented in the Appendix A.1.

of agents. Second, asset holdings appear in the Euler equation. However, in the two-economy case this includes not only domestic, but also foreign assets. As a result financial asset holdings exert an impact on agents decisions, breaking not only Ricardian equivalence, but also its open-economy counterpart. Our interpretation is that behavioral households accumulate additional domestic and foreign assets (as compared with rational agents), as they myopically underestimate future income. This has a bearing on current-period consumption. For instance, if future income is expected to deviate positively from the steady state, behavioral agents expect a smaller increase than rational agents and respond with raising their consumption by less than rational households would have done. In general equilibrium this results in additional asset holdings, that would not materialize under RE.

Furthermore we derive an uncovered interest parity (UIP) condition, modified for myopic behavior (as compared with the RE counterpart)

$$\underbrace{\hat{R}_t - mE_t\hat{\pi}_{t+1}}_{\substack{\text{behaviorally expected domestic} \\ \text{real interest rate}}} = \underbrace{\hat{R}_t^* + \hat{\kappa}_t - m\hat{\pi}_{t+1}^*}_{\substack{\text{behaviorally expected foreign} \\ \text{real interest rate}}} + \underbrace{mE_t\hat{q}_{t+1} - \hat{q}_t}_{\substack{\text{behaviorally expected} \\ \text{RER depreciation rate}}} \quad (5)$$

where $q_t \equiv S_t P_t^*/P_t$ denotes the real exchange rate (RER). Behavioral agents discount the deviations of the expected exchange rate and inflation rates from the steady state. Note that the equilibrium conditions collapse to standard ones with the behavioral parameter m set to unity. Hence the model nests the standard RE benchmark as a special case.

3.3 Producers

The production process takes two steps. Final consumption good producers use intermediate goods of domestic and foreign variety to manufacture their product. Monopolistically competitive intermediate goods producers employ labor to produce differentiated goods for both domestic and foreign markets. All firms are owned by households.

3.3.1 Consumption good producers

Perfectly competitive final consumption good producers purchase domestic and foreign varieties of differentiated intermediate goods $y_H(i)$ and $y_F(i)$ to produce a homogeneous good according to the following technology

$$y_t = \left((1 - \eta_H)^{\frac{1}{\phi_y}} y_{F,t}^{\frac{\phi_y - 1}{\phi_y}} + \eta_H^{\frac{1}{\phi_y}} y_{H,t}^{\frac{\phi_y - 1}{\phi_y}} \right)^{\frac{\phi_y}{\phi_y - 1}} \quad (6)$$

where

$$y_{H,t} = \left(\int_0^1 y_{H,t}(i)^{\frac{1}{\varepsilon_{p,t}}} di \right)^{\varepsilon_{p,t}} \quad (7)$$

$$y_{F,t} = \left(\int_0^1 y_{F,t}(i)^{\frac{1}{\varepsilon_{p,t}}} di \right)^{\varepsilon_{p,t}} \quad (8)$$

In the formulas above, η_H determines the home bias in consumption, ϕ_y is the elasticity of substitution between domestic and foreign goods, while $\varepsilon_{p,t}$ denotes the price mark-up (which depends on the elasticity of substitution between differentiated intermediate goods). The markup is assumed to follow an AR(1) process with mean ε_p , persistence ρ_p and standard deviation of innovations σ_p .

From this problem we obtain the demand for intermediate good varieties $y_{H,t}(i)$ and $y_{F,t}(i)$

$$y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{\frac{-\varepsilon_{p,t}}{\varepsilon_{p,t}-1}} y_{H,t} \quad (9)$$

$$y_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{\frac{-\varepsilon_{p,t}}{\varepsilon_{p,t}-1}} y_{F,t} \quad (10)$$

where $P_{H,t}$ denotes the price of the home variety good and $P_{F,t}$ of the foreign variety good.

3.3.2 Intermediate goods producers

Intermediate goods producers, indexed by i , use the following technology

$$y_{H,t}(i) + \frac{1-\omega}{\omega} y_{H,t}^*(i) = z n_t(i) \quad (11)$$

This production function gives rise to the formula determining marginal costs

$$mc_t = \frac{w_t}{z}$$

where $w_t \equiv W_t/P_t$.

Next, we specify the firm problem. First, notice that the marginal cost is constant and independent of the production volume. Therefore, we can separate the firm problem into the one in the domestic market and the one in the foreign market. Intermediate goods producers operate in a monopolistically competitive environment and set their prices according to the Calvo scheme. In each period, a producer i receives with probability $1 - \theta_H$ a signal to reoptimize her price. Otherwise, prices are indexed according to $\pi_t^{\zeta,H} = \zeta_H \pi_{t-1} + (1 - \zeta) \bar{\pi}$, where ζ_H controls the degree of indexation to past inflation. It is also convenient to denote inflation between periods t and $t + j$ as $\pi_{t,t+j} = \pi_{t+1} \cdot \pi_{t+2} \cdot \dots \cdot \pi_{t+j}$ as well as indexation between periods t and $t + j$ as $\pi_{t,t+j}^{\zeta,H} = \pi_{t+1}^{\zeta,H} \cdot \pi_{t+2}^{\zeta,H} \cdot \dots \cdot \pi_{t+j}^{\zeta,H}$. Let also $\Lambda_{t,t+j} = \frac{c_{t+j}^{-\sigma}}{c_t^{-\sigma}}$ be the

stochastic discount factor. In period t the producer who obtained the Calvo signal sets the price $\tilde{P}_{H,t}$ to maximize the following problem:

$$\max_{\tilde{P}_{H,t}(i), \{y_{H,t+j}(i)\}_{j=0}^{\infty}} E_t^{BE} \sum_{j=0}^{\infty} (\beta\theta_H)^j \Lambda_{t,t+j} \left(\frac{\tilde{P}_{H,t}(i) \pi_{t,t+j}^{\zeta,H}}{P_{t+j}} - mc_{t+j} \right) y_{H,t+j}(i) \quad (12)$$

subject to the demand function given by modified equation (9)

$$y_{H,t+j}(i) = \left(\frac{\tilde{P}_{H,t}(i) \pi_{t,t+j}^{\zeta,H}}{P_{H,t+j}} \right)^{\frac{-\varepsilon_{p,t}}{\varepsilon_{p,t}-1}} y_{H,t+j} \quad (13)$$

Following Gabaix (2020), we solve the problem and obtain the following behavioral Phillips curve

$$\begin{aligned} \theta_H(\hat{p}_{H,t} - \hat{p}_{H,t-1} + \hat{\pi}_t - \zeta_H \hat{\pi}_{t-1}) = \\ (1 - \theta_H)(1 - \beta\theta_H)(\hat{m}c_t - \hat{p}_{H,t} + \hat{\varepsilon}_{p,t}) - (1 - \theta_H)\beta\theta_H(1 - m)(\hat{p}_{H,t} - \zeta_H \hat{\pi}_t) \\ + \beta\theta_H m(E_t \hat{p}_{H,t+1} - \hat{p}_{H,t} + E_t \hat{\pi}_{t+1} - \zeta_H \hat{\pi}_t) \end{aligned} \quad (14)$$

Note that it implies that under behavioral expectations price setters are more responsive to current variables (like marginal cost) and less responsive to expected inflation as compared to rational agents.

The problem of the home good producer producing for the foreign market is analogous with the exception that prices are expressed and sticky in the foreign currency. Solving it results in the following behavioral Phillips curve

$$\begin{aligned} \theta_H^*(\hat{p}_{H,t}^* - \hat{p}_{H,t-1}^* + \hat{\pi}_t^* - \zeta_H^* \hat{\pi}_{t-1}^*) = \\ (1 - \theta_H^*)(1 - \beta\theta_H^*)(\hat{m}c_t - \hat{q}_t - \hat{p}_{H,t}^* + \hat{\varepsilon}_{p,t}^*) - (1 - \theta_H^*)\beta\theta_H^*(1 - m)(\hat{p}_{H,t}^* - \zeta_H^* \hat{\pi}_t^*) \\ + \beta\theta_H^* m(E_t \hat{p}_{H,t+1}^* - \hat{p}_{H,t}^* + E_t \hat{\pi}_{t+1}^* - \zeta_H^* \hat{\pi}_t^*) \end{aligned} \quad (15)$$

Finally, the zero profit condition for the final good producer implies

$$\eta \hat{p}_{H,t} + (1 - \eta) \hat{p}_{F,t} = 0 \quad (16)$$

3.4 Government and the central bank

The government uses lump sum taxes to finance government expenditure. The government's budget constraint in this economy is given by

$$g_t + R_t b_{G,t-1} = T_t + b_{G,t}. \quad (17)$$

where $b_{G,t} = B_{G,t}/P_t$ denotes the real value of government debt $B_{G,t}$. We assume that the fiscal rule on taxes is given by the following formula

$$\frac{T_t - \bar{T}}{\bar{T}} = \nu \frac{b_{G,t} - \bar{b}_G}{\bar{b}_G}$$

Government expenditures are assumed constant

$$g_t = \bar{g} \tag{18}$$

We assume that the central bank sets the policy rate according to the following Taylor rule responding to GDP and inflation

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\gamma_\pi} \left(\frac{\tilde{y}_t}{\bar{y}} \right)^{\gamma_y} \right]^{1-\gamma_R} e^{\varepsilon_{R,t}} \tag{19}$$

where γ_π and γ_y control the strength of policy rate response to inflation and output respectively, while γ_R controls the degree of interest rate smoothing. \tilde{y} denotes GDP and $\varepsilon_{R,t}$ is an i.i.d. monetary policy shock with standard deviation σ_R .

3.5 Closing the model

3.5.1 GDP and Balance of Payments

We define aggregate output (GDP) in the standard fashion

$$\tilde{y}_t \equiv c_t + g_t + nx_t \tag{20}$$

where net exports nx_t are given by

$$nx_t = q_t p_{H,t}^* \frac{1-\omega}{\omega} y_{H,t}^* - p_{F,t} y_{F,t}$$

The balance of payments can be written as

$$b_{H,t}^* = nx_t + \frac{q_t}{q_{t-1} \pi_t^*} b_{H,t-1}^* R_{t-1}^* \rho_{t-1} \tag{21}$$

3.5.2 Market clearing

We impose a standard set of market clearing conditions. Final good market clearing implies

$$c_t + g_t = y_t \tag{22}$$

The labor market clears when

$$\int_0^1 n_t(i) di = n_t \quad (23)$$

Finally, the markets for bonds clear as well

$$\omega B_{H,t} + (1 - \omega) B_{F,t} = \omega B_{G,t} \quad (24)$$

The full set of linearized equilibrium conditions is presented in Appendix A.3.

4 Calibration and estimation

4.1 Calibrated parameters

As it is standard in the literature, we calibrate most of the parameters affecting the model's steady state equilibrium. We use as targets the averages of key macroeconomic proportions observed in the data over the period 1999-2021. The values of all calibrated parameters are presented in Table 1 and the targeted steady state ratios are reported in Table 2.

Our model features two countries: Poland and the euro area (EA). We calibrate the size of Poland to 5%, which roughly corresponds to its share in total GDP. We set the share of home-made goods in Poland's consumption basket at 0.7, which is consistent with the estimates of Bussière et al. (2011). Adjusting this number for the relative size of the two countries leads to the corresponding share in the euro area of 0.98.

To calibrate the steady state shares of government debt we calculate the average GDP ratios of government debt held by domestic agents in Poland and in the euro area. The numbers are 31.7% and 48.4% in annual terms, respectively. Poland's steady state foreign debt-to-GDP ratio is calibrated to the net (i.e. liabilities minus assets) debt securities held by domestic agents according to the international investment position statistics. The calibrated debt is 15% in annual terms.

Since in the data we neither see strong evidence of long-term differences between Poland and the euro area as regards the remaining steady state ratios, nor the observed heterogeneity is important for our main results, we keep the rest of our calibration symmetric across the two regions. The share of government spending in GDP is fixed at 0.25. As in Brzoza-Brzezina et al. (2014), we assume that the elasticity of substitution between good varieties equals 3 (implying an average gross markup of 1.33). We set the discount factors to 0.995 to match an annual real interest rate of 2% and fix the inverse of the Frisch elasticity for labor supply at 1.

4.2 Data and prior assumptions

We estimate the remaining parameters using time series covering the period 1999q1-2022q2, giving us 94 quarterly observations. The starting point coincides with the introduction of the euro and adoption of inflation targeting in Poland. We use the following three pairs of data series for the the EA and Poland: real private consumption, HICP inflation and the short-term interest rate. We also treat as observable the real exchange rate between Poland and the EA.

To estimate the model we use Bayesian methods. Our priors of most parameters are based on the previous literature. For the behavioral parameter m we assume a beta prior with mean 0.85, a number proposed by Gabaix (2020).

Our model is driven by seven stochastic shocks. These include the pairs of time preference, price markup, and monetary policy shocks in both economies and an international risk premium shock. All shocks are modeled as first-order autoregressive processes, except for the monetary policy shock that is assumed to be white noise. We set the prior means of shock inertia to 0.7, with fairly large standard deviations. The prior distributions of shock volatilities are centered around 0.01. A smaller prior mean of 0.0025 is assumed for the monetary policy shocks, consistently with the previous literature.

4.3 Estimation results

We estimate the model using Dynare version 4.5.7. The posterior mode in the first pass is obtained with Christopher Sims' optimization routine. We next run the Metropolis-Hastings algorithm with two blocks, each consisting of 250,000 replications. Convergence was confirmed by a set of diagnostic tests proposed by Brooks and Gelman (1998). Finally, the posterior distributions are approximated using the second half of the draws. The posterior moments are reported in Tables 3 and 4. Our dataset is informative about all of the estimated parameters. The posterior modes are broadly consistent with earlier estimates in the literature obtained for the euro area and Poland.

On top of our BE model we also estimate a model that features rational expectations. We assume the same prior distributions, the only exception being the behavioral parameter, which we calibrate to unity. The goal of this exercise is to compare how well the data fits the two frameworks. To this end we use the standard tool of Bayesian model comparison and compare the marginal log-data densities of the two models. These are 2324 and 2305 for the BE and RE models respectively, giving a posterior odds ratio of $1.7 \cdot 10^8$. Assuming equal prior model probabilities this result points towards the behavioral model being much more likely than the rational expectation model. To check if the BE model marginal likelihood does not depend on the prior assumption we also estimated it with m fixed at the previously estimated value (i.e. $m = 0.71$). The obtained marginal log-data density of this model is 2328, which gives us an even higher odds ratio (in comparison to the RE model) of $8.3 \cdot 10^9$. We conclude that the data clearly favors the BE framework and, in what follows, we will treat it as the benchmark model.

Estimating both the BE and RE model allows to take a deeper look at an important and interesting question: which features of the framework are affected by allowing agents to act behaviorally? The last columns of tables 3 and 4 present the posterior means of parameters estimated under RE. Comparing these to the BE model shows that structural (so called deep) parameters remain mainly untouched. What changes between the two models are processes driving the stochastic components. In particular in the BE model standard deviations and autoregression coefficients of preference shocks and the risk premium shocks are substantially smaller. In other words, the BE model relies less on exogenous forces and more on internal propagation to explain the data. This should be seen as a welcome feature.

5 Consequences of behavioral assumptions for monetary policy

Let us now move to discussing our main findings. We begin with discussing the consequences of agents' behavioral features for monetary policy. In the next section we move to the more general questions, in particular an assessment whether BE can help solve selected puzzles in open economy macroeconomics.

We start with describing the working of monetary policy in our framework and comparing it with the RE benchmark. Then we focus on the following questions: (i) how is the effectiveness of monetary policy affected by the behavioral/rational nature of agents? (ii) does the monetary policy trade-off change? (iii) how does the behavioral nature of agents affect the determinacy principles for monetary policy?

5.1 The working of monetary policy

We begin with comparing how monetary policy affects the economy in a world with behavioral and rational agents. To this end we plot impulse response functions to a contractionary monetary policy shock for three variants of our model: the estimated model with behavioral agents, the estimated model with rational agents and a counterfactual model estimated with behavioral agents and a counterfactual assumption imposed ex post that $m=1$. The first model is the benchmark according to our Bayesian model comparison exercise. The second model is the “wrong” one according to the same criterion, but it is the kind of model frequently estimated by economists and used by policymakers. Its properties allow to answer the question what an economist would believe about the economy if he/she sticks to the RE assumption. In contrast, the last model offers an answer about a counterfactual world. It shows how monetary policy would work if agents became rational (instead of being behavioral).

Figures 1 and 2 show the impulse responses for the small and large economy respectively. The main conclusions seem not to depend to a large extent on the size and openness of the economy. Under behavioral expectations contractionary monetary policy lowers output and inflation in the usual fashion (blue solid line). If instead of being behavioral agents were rational, monetary policy would be much more powerful (counterfactual scenario, yellow dotted line). This happens, since behavioral agents take future developments (and hence the persistently higher interest rate) into account to a limited extent. To check this intuition we counterfactually assumed the persistence parameter $\gamma_R = 0$. In this case impulse responses differ between the BE and RE model only slightly. The last (red dashed) line shows that estimating the wrong model (with RE instead of BE) can lead to quite substantial mistakes in the assessment of the effectiveness of monetary policy. Under the wrong model monetary policy seems more powerful in the short run than the benchmark model indicates. As mentioned, all these conclusions hold both for the small and large economy.

5.2 The monetary policy trade-off

The implications of behavioral expectations for the effectiveness of monetary policy described above were derived for a given parametrization of the monetary policy rule. Now we go one step further and look at a wider set of policies. In particular, we optimize the behavior of monetary policy and analyze the trade-off it faces. Following much of the literature, the Taylor rule given by (9) is assumed to respond to deviation from the steady state of both output \tilde{y}_t and inflation π_t with the parameters denoting the respective strength of response γ_y and γ_π .

We study the trade-off between stabilizing these two variables. To this goal we construct an efficient policy frontier by finding aforementioned coefficients of the Taylor rule that minimize a simple loss function with a full spectrum of weights on output vs. inflation stabilization. More specifically, to obtain the monetary policy frontier we solve the following set of minimization problems indexed by $\lambda \in [0, 1]$

$$\min_{\gamma_\pi, \gamma_y} \{ \lambda D(\pi_t) + (1 - \lambda) D(\tilde{y}_t) \}$$

where $D(x_t)$ denotes the ergodic standard deviation of x_t .

The obtained policy frontiers for both the small and large country are presented, respectively, on Figures 3 and 4 by plotting the standard deviation of GDP against that of inflation. The blue line represents the estimated model with behavioral expectations, the red one the same model but with behavioral parameter m set to 0.8, the yellow one the model with m set to 0.9, and the violet one with m set to 1 (thus representing the case with rational agents).

One result that stands out corresponds to the slope of the frontiers. This is the sacrifice ratio in our economy, as it shows how much of GDP volatility needs to be sacrificed in order to lower the inflation volatility by 1 p.p. Both figures show that as the degree of cognitive discounting increases the sacrifice ratio goes up. Stabilizing inflation with behavioral agents is thus more costly, both in the small and large economy (though the impact of cognitive discounting is stronger in the SE).⁴ Intuitively, it follows from the fact that price setters respond less to expected inflation than in case of rational expectations. It means that the expectation channel of monetary policy is weaker and, as a result, monetary policy needs to generate larger movements of GDP in order to stabilize inflation. Therefore, lowering inflation volatility is more costly in terms of GDP volatility.

⁴In order to ensure that the latter finding depends indeed on the size of the economies we ran an additional simulation with all parameters but size set symmetrically to prior means (as in Section 5.3).

5.3 Determinacy of monetary policy

Stabilizing properties of monetary policy rules have been widely discussed in the economic literature, both in the context of the New Keynesian modeling framework (Bullard and Mitra, 2002) and of historical developments (Sargent et al., 2006). It is well known that in the New Keynesian framework, on which we build, the Taylor principle states that the reaction of the interest rate to inflation ought to be strictly larger than one. However, as shown in Gabaix (2020), under the behavioral assumption used in this paper the stability conditions are less strong, and reactions (somewhat) below unity can also ensure determinacy.

In what follows we check whether this finding goes through in the open-economy behavioral environment and whether the Taylor principle differs for small and large economies. To make the exercise as telling as possible about the role of economic openness, we make an exception and calibrate the two economies completely symmetric (except for size and related openness of course). To this end we set all debt ratios to zero, and set all structural parameters to their prior mean values. Additionally, to concentrate on the role of γ_π and γ_π^* we set the remaining Taylor rule parameters γ_y , γ_y^* , γ_R and γ_R^* to zero.

With such calibration we check the determinacy boundaries. On Figure 5 we plot the boundary values for γ_π and γ_π^* as functions of m (each time fixing the reaction to inflation in the other economy at 1.25). The stability regions clearly increase as agents become more behavioral, but interestingly this effect is stronger in the small country. For instance for $m = 0.85$ the monetary policy can be passive, with reaction in the SE as weak as 0.03, while in the LE the central bank has to respond to inflation with $\gamma_\pi^* > 0.56$. Of course, as is well known, under rational expectations the boundary is unity independently of the size of the economy.

Two questions emerge. First, why are stability regions larger under BE than under RE? Our understanding is that this is the consequence of the steady state being an attractor for agents expectations to a larger extent than under RE. As expectations affect decisions (in particular pricing decisions), this mechanism has a stabilizing property that partly substitutes for the weaker reaction of the central bank.

Second, why are stability regions larger in the SE? The explanation above sheds light on this puzzle. Under BE also the expectations of the exchange rate are attracted towards the steady state. This generates an additional stabilization mechanism for import prices. However, given the different openness of the two economies, this additional stabilizing force is much more important for the small economy, hence its larger determinacy regions.

6 Behavioral agents and open-economy puzzles

There is a number of findings related to open economies where empirical evidence and structural models do not align. Does incorporation of the behavioral nature of agents help to bring models closer to reality? In what follows we concentrate on two issues, where BE improve upon the RE framework. The first is the well-known uncovered interest rate parity (UIP) puzzle identified by Fama (1984). Then we discuss the disconnect between modeling outcomes and empirical findings regarding the independence of monetary policy under flexible exchange rates.

6.1 Excess returns (UIP puzzle)

One of the important puzzles in macroeconomics is related to the behavior of the exchange rate (Fama 1984; Engel 2013). Empirical evidence shows that after an unexpected increase of the domestic interest rate positive excess returns can be achieved on the domestic currency (or negative on the foreign one) for several periods. This stands in sharp contrast with the conclusion found under rational expectations, that after an interest rate shock the exchange rate adjusts immediately so that further gains on the domestic bond are exactly matched by expected exchange rate depreciation and, hence expected excess returns are zero.

Let us define excess returns (on the foreign currency):

$$XR_{t+1} \equiv (R_t^f - R_t) + (S_{t+1} - S_t) \quad (25)$$

where R_t^f denotes the foreign interest rate. Following Fama (1984) the empirical literature usually estimates an equation like:

$$XR_{t+k} = a + b(R_t - R_t^f) + \epsilon_t \quad k = 1, 2, \dots \quad (26)$$

where a and b are estimated parameters. If UIP holds, both estimated coefficients should be zero at any horizon k . However, as mentioned, empirical studies usually find substantial deviations from UIP, in particular with b being negative, suggesting that excess returns can be achieved systematically. For instance Candian and DeLeo (2022) report a panel estimate of b to rise from approximately -2 for $k = 1$ to zero for $k = 20$ (and then becoming positive for some time). Burnside et al. (2006) report negative values of b in the 1 and 3 month horizons for nine advanced economies.

Let us now check how things look like in our estimated model. As was shown in Section 3 under BE the UIP condition is modified, which suggests that the relation between interest rates and the exchange rate may differ from the rational world (see also Kolasa et al. (2022)

for analytical results). Figure 6 shows how the exchange rate, excess return⁵ and expected excess return behave after a domestic monetary policy shock that raises the interest rate differential. Clearly, under RE excess returns are achieved only on impact (as interest rates increase by surprise). Expected excess returns are hence always zero. In contrast, under BE excess returns exist for a number of quarters after the shock and in particular they can be forecasted (assuming an agent forecasts rationally).

To offer a quantitative comparison with the empirical literature we simulate data from our model and estimate⁶ equation (26) for $k = 1, \dots, 20$. Figure 7 shows the estimated b coefficients for the BE and RE models. As can be expected under rational expectations b is never significantly different from zero. In contrast, under BE the coefficients are negative rising from -0.5 on impact to zero after approximately 3 years. So, the effect seems of magnitude comparable to, though somewhat weaker than, documented in empirical studies.

6.2 International monetary policy independence

Standard macroeconomic theory states that under flexible exchange rates and liberalized international capital flows monetary policy in a small open economy is insulated from foreign monetary policy shocks (Obstfeld et al., 2005). However, in practice monetary policy rates in small open economies usually track interest rates in larger countries, even when the exchange rate is allowed to fluctuate freely. Of course, this might be the result of common cyclical fluctuations, but several research studies have documented that interest rates co-move also as a consequence of exogenous monetary policy shocks in the large economy (see e.g. Frankel et al. (2004), Borensztein et al., 2001 and Moder (2021) for an international comparison and Crespo Cuaresma and Wojcik (2006) and Goczek and Mycielska (2019) for Poland).

In what follows we check how a monetary policy shock in the large economy affects interest rates in the small economy. Figure 8 documents our findings. Under rational expectations (whether estimated or counterfactual) the SE interest rate declines in response to a monetary policy tightening abroad. This seems inconsistent with the empirical evidence cited above. However, under behavioral expectation monetary policy in the SE is tightened, in line with empirical evidence. This is due to the inflation reaction being much spread over time (in contrast to the very sharp decline under RE). It should, however be admitted that the effect is relatively weak, so that in spite of moving in the right direction its quantitative relevance is not sufficient to explain the puzzle in whole.⁷

⁵Since our model features a well-defined risk premium, we include it in the definition of the foreign interest rate, so that $R_t^f \equiv R_t^* + \kappa_t$.

⁶We generate 1000 simulations of 200 period long samples

⁷For instance Borensztein et al., 2001 and Moder (2021) report elasticities of domestic to foreign (US or EA) interest rates in selected flexible exchange rate countries of 1/3 to 2/3. In contrast, in our BE case the elasticity hardly rises to 1/10 in the long run.

7 Robustness

How robust are our findings? We answer this question by performing an additional estimation of our model. As we stated above in our baseline estimation we decided to use data that includes the COVID-19 period. However, as the pandemic period was highly volatile, we decided to check if such an economic shock does not change the conclusions drawn from our model. To check if that is the case, we estimate our model with data that end in 4q2019, just before the COVID-19 outbreak. Moreover, in our baseline estimation, we use the EA short-term interest rate. An alternative is to use a shadow rate which may be a better reflection of the monetary policy stance in the Eurozone during the ZLB period. To check also such a case, we also use the shadow rate taken from Wu and Xia (2020).

The posterior distribution of estimated parameters is not significantly different from that of our baseline estimation. The differences can be found in the shock standard deviations, which is not surprising, as we cut a significant part of volatility from the data by not considering the COVID-19 period. The conclusions drawn from simulations that were repeated using the second estimation are consistent with those based on the baseline dataset.

8 Conclusions

Since the seminal papers of Muth (1961); Lucas (1972) and Kydland and Prescott (1982) the rational expectations assumption has dominated in the macroeconomics literature. However, there is mounting evidence that agents do not always act rationally. Moreover, an increasing number of papers documents that deviations from rationality may exert a significant impact on macroeconomic processes. In this paper we consider a two-economy macroeconomic business cycle model with behavioral expectations a la Gabaix (2020) and use it to take a fresh look at several important topics related to monetary policy in the two-economy framework. Not only do our findings confirm the importance of the behavioral expectations assumption in the open-economy context, but they also have the potential to change the way we think about the effectiveness of monetary policy and the alignment of the standard open-economy framework with empirical regularities.

First, model estimation on data for Poland and the euro area confirms that the assumption about agents acting in a behavioral fashion is strongly preferred by the data.

Second, we derive a number of conclusions about monetary policy. Under behavioral expectations determinacy regions for monetary policy are substantially larger than under rational expectations (passive policy can also ensure determinacy) and additionally depend on the size of the economy. Our novel finding is that stability regions are larger for small (more open) than for large economies. This draws back to the altered behavior of the expected exchange rate which acts as an additional stabilizing device in the small economy. Third, with agents acting in a behavioral manner the monetary policy trade-off worsens. The sacrifice ratio increases both in the small and large economy, so that the real cost of stabilizing inflation is higher, but the effect is stronger for small economies.

Moreover, our framework delivers some interesting findings related to the performance and properties of microfounded open-economy macro models. As is well known these models generate a number of puzzling effects, in particular the lack of excess returns (UIP puzzle) and a strong independence of monetary policy in small economies from that in large economies. We check whether our estimated behavioral framework can contribute to solving these puzzles and report partial success. Excess returns exist under BE in line with empirical evidence. Monetary policy shocks in the large economy leads to an equally-signed reaction in the small economy (though weaker than suggested by empirical evidence).

All in all, the open-economy behavioral framework is strongly supported by the data and delivers a number of findings which both policymakers and modelers should find interesting and important.

References

- Bacchetta, Philippe, and Eric Van Wincoop (2010) ‘Infrequent portfolio decisions: A solution to the forward discount puzzle.’ *American Economic Review* 100(3), 870–904
- Baron, Matthew, and Wei Xiong (2017) ‘Credit expansion and neglected crash risk.’ *The Quarterly Journal of Economics* 132(2), 713–764
- Benchimol, Jonathan, and Lahcen Bounader (2021) ‘Optimal Monetary Policy Under Bounded Rationality.’ Dynare Working Papers 67
- Bordalo, Pedro, Nicola Gennaioli, Andrei Shleifer, and Stephen J Terry (2021) ‘Real credit cycles.’ Technical Report, National Bureau of Economic Research
- Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer (2020) ‘Overreaction in macroeconomic expectations.’ *American Economic Review* 110(9), 2748–82
- Borensztein, Eduardo, Thomas Philippon, and Jeromin Zettelmeyer (2001) ‘Monetary Independence in Emerging Markets: Does the Exchange Rate Regime Make a Difference?’ IMF Working Papers 2001/001, International Monetary Fund, January
- Bounader, Lahcen, and Guido Traficante (2022) ‘Robustly optimal monetary policy in a behavioral environment.’ *Available at SSRN 4011146*
- Brooks, Stephen, and Andrew Gelman (1998) ‘Some issues in monitoring convergence of iterative simulations.’ In ‘Proceedings of the Section on Statistical Computing’ American Statistical Association.
- Brzoza-Brzezina, Michał, Pascal Jacquinot, and Marcin Kolasa (2014) ‘Can We Prevent Boom-Bust Cycles During Euro Area Accession?’ *Open Economies Review* 25(1), 35–69
- Bullard, James, and Kaushik Mitra (2002) ‘Learning about monetary policy rules.’ *Journal of Monetary Economics* 49(6), 1105–1129
- Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo (2006) ‘The Returns to Currency Speculation.’ Technical Report
- Bussière, Matthieu, Giovanni Callegari, Fabio Ghironi, Giulia Sestieri, and Norihiko Yamano (2011) ‘Estimating trade elasticities: Demand composition and the trade collapse of 2008–09.’ NBER Working Papers 17712, National Bureau of Economic Research, Inc
- Calvo, Guillermo (1983) ‘Staggered prices and in a utility-maximizing framework.’ *Journal of Monetary Economics* 12(3), pp. 383–398

- Candian, Giacomo, and Pierre DeLeo (2022) ‘Imperfect Exchange Rate Expectations.’ mimeo, mimeo, HEC Montreal
- Coibion, Olivier, and Yuriy Gorodnichenko (2015) ‘Information rigidity and the expectations formation process: A simple framework and new facts.’ *American Economic Review* 105(8), 2644–78
- Crespo Cuaresma, Jesus, and Cezary Wojcik (2006) ‘Measuring monetary independence: Evidence from a group of new EU member countries.’ *Journal of Comparative Economics* 34(1), 24–43
- Del Negro, Marco, Marc Giannoni, and Christina Patterson (2012) ‘The forward guidance puzzle.’ Staff Reports 574, Federal Reserve Bank of New York
- Engel, Charles (2013) ‘Exchange Rates and Interest Parity.’ NBER Working Papers 19336, National Bureau of Economic Research, Inc, August
- Eusepi, Stefano, and Bruce Preston (2011) ‘Expectations, learning, and business cycle fluctuations.’ *American Economic Review* 101(6), 2844–72
- Evans, George W, and Seppo Honkapohja (1999) ‘Learning dynamics.’ *Handbook of macroeconomics* 1, 449–542
- Fahlenbrach, Rüdiger, Angie Low, and René M Stulz (2017) ‘Do independent director departures predict future bad events?’ *The Review of Financial Studies* 30(7), 2313–2358
- Fama, Eugene F. (1984) ‘Forward and spot exchange rates.’ *Journal of Monetary Economics* 14(3), 319–338
- Frankel, Jeffrey, Sergio L. Schmukler, and Luis Servén (2004) ‘Global transmission of interest rates: monetary independence and currency regime.’ *Journal of International Money and Finance* 23(5), 701–733
- Gabaix, Xavier (2019) ‘Behavioral inattention.’ In ‘Handbook of Behavioral Economics: Applications and Foundations 1,’ vol. 2 (Elsevier) pp. 261–343
- Gabaix, Xavier (2020) ‘A Behavioral New Keynesian Model.’ *American Economic Review* 110(8), 2271–2327
- Gabaix, Xavier, and David Laibson (2017) ‘Myopia and discounting.’ Technical Report, National bureau of economic research

-
- Gali, Jordi (2020) ‘Uncovered interest parity, forward guidance and the exchange rate.’ *Journal of Money, Credit and Banking* 52(S2), 465–496
- García-Schmidt, Mariana, and Michael Woodford (2019) ‘Are low interest rates deflationary? a paradox of perfect-foresight analysis.’ *American Economic Review* 109(1), 86–120
- Gennaioli, Nicola, and Andrei Shleifer (2018) ‘A crisis of beliefs.’ In ‘A Crisis of Beliefs’ (Princeton University Press)
- Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny (2015) ‘Neglected risks: The psychology of financial crises.’ *American Economic Review* 105(5), 310–14
- Goczek, Lukasz, and Dagmara Mycielska (2019) ‘Actual monetary policy independence in a small open economy: the Polish perspective.’ *Empirical Economics* 56(2), 499–522
- Greenwood, Robin, and Samuel G Hanson (2013) ‘Issuer quality and corporate bond returns.’ *The Review of Financial Studies* 26(6), 1483–1525
- Greenwood, Robin, Samuel G Hanson, Andrei Shleifer, and Jakob Ahm Sørensen (2020) ‘Predictable financial crises.’ Technical Report, National Bureau of Economic Research
- Hommel, Cars, Domenico Massaro, and Matthias Weber (2019) ‘Monetary policy under behavioral expectations: Theory and experiment.’ *European Economic Review* 118(C), 193–212
- Jordà, Òscar, Moritz Schularick, and Alan M Taylor (2015) ‘Leveraged bubbles.’ *Journal of Monetary Economics* 76, S1–S20
- Kahneman, Daniel, and Amos Tversky (1972) ‘Subjective probability: A judgment of representativeness.’ *Cognitive psychology* 3(3), 430–454
- Kolasa, Marcin, Pawel Zabczyk, and Sahil Ravgotra (2022) ‘Monetary Policy and Exchange Rate Dynamics in a Behavioral Open Economy Model.’ IMF Working Papers 2022/112, International Monetary Fund, June
- Kydland, Finn E, and Edward C Prescott (1982) ‘Time to build and aggregate fluctuations.’ *Econometrica: Journal of the Econometric Society* pp. 1345–1370
- L’Huillier, Jean-Paul, Sanjay R Singh, and Donghoon Yoo (2021) ‘Incorporating diagnostic expectations into the new keynesian framework.’ Available at SSRN 3910318
- López-Salido, David, Jeremy C Stein, and Egon Zakrajšek (2017) ‘Credit-market sentiment and the business cycle.’ *The Quarterly Journal of Economics* 132(3), 1373–1426

- Lucas, Robert E (1972) ‘Expectations and the neutrality of money.’ *Journal of economic theory* 4(2), 103–124
- Maćkowiak, Bartosz, and Mirko Wiederholt (2015) ‘Business cycle dynamics under rational inattention.’ *The Review of Economic Studies* 82(4), 1502–1532
- Moder, Isabella (2021) ‘The transmission of euro area monetary policy to financially euroised countries.’ Working Paper Series 2611, European Central Bank, October
- Muth, John F (1961) ‘Rational expectations and the theory of price movements.’ *Econometrica: Journal of the Econometric Society* pp. 315–335
- Obstfeld, Maurice, Jay C. Shambaugh, and Alan M. Taylor (2005) ‘The Trilemma in History: Tradeoffs Among Exchange Rates, Monetary Policies, and Capital Mobility.’ *The Review of Economics and Statistics* 87(3), 423–438
- Pintus, Patrick A, and Jacek Suda (2019) ‘Learning financial shocks and the great recession.’ *Review of Economic Dynamics* 31, 123–146
- Sargent, Thomas, Noah Williams, and Tao Zha (2006) ‘Shocks and government beliefs: The rise and fall of american inflation.’ *The American Economic Review* 96(4), 1193–1224
- Schmitt-Grohe, Stephanie, and Martin Uribe (2003) ‘Closing small open economy models.’ *Journal of International Economics* 61(1), 163–185.
- Tversky, Amos, and Daniel Kahneman (1974) ‘Judgment under uncertainty: Heuristics and biases: Biases in judgments reveal some heuristics of thinking under uncertainty.’ *science* 185(4157), 1124–1131
- Woodford, Michael (2019) ‘Monetary policy analysis when planning horizons are finite.’ *NBER Macroeconomics Annual* 33(1), 1–50
- Wu, Jing Cynthia, and Fan Dora Xia (2020) ‘Negative interest rate policy and the yield curve.’ *Journal of Applied Econometrics* 35(6), 653–672

Tables and figures

Table 1: Calibration - parameters

Parameter	Value	Description
ω	0.05	Agents residing in the small economy (Poland)
β, β^*	0.995	Discount factors
γ, γ^*	1	Inverse of Frisch elasticity of labor supply
$\varepsilon_p, \varepsilon_p^*$	3	Elasticity of substitution between good varieties
η_H	0.7	Home bias in consumption (Poland)
η_H^*	0.98	Home bias in consumption (EA)
ν, ν^*	0.1	Semi-elasticity of taxes

Table 2: Steady state ratios

Steady state ratio	Value
Export to GDP ratio (SE)	0.30
Export to GDP ratio (LE)	0.016
Government spending to GDP ratio	0.25
Gov. debt to GDP ratio (Poland)	1.27
Gov. debt to GDP ratio (Euro area)	1.74
Foreign debt to GDP ratio (Poland)	0.60

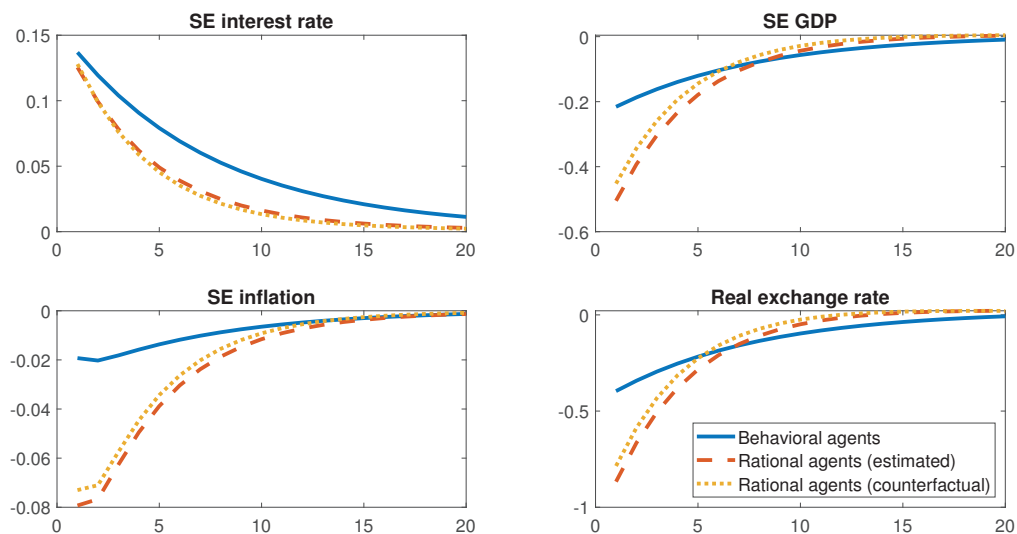
Table 3: Prior and posterior distribution: structural parameters

Parameter	Prior distribution			BE posterior distribution		RE posterior distribution	
	type	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
m	beta	0.85	0.05	0.71	0.04	-	-
θ_h	beta	0.75	0.1	0.89	0.03	0.88	0.03
θ_h^*	beta	0.75	0.1	0.83	0.7	0.89	0.6
θ_f	beta	0.75	0.1	0.86	0.03	0.87	0.03
θ_f^*	beta	0.75	0.1	0.89	0.02	0.90	0.02
ζ_h	beta	0.25	0.1	0.17	0.07	0.19	0.08
ζ_h^*	beta	0.25	0.1	0.24	0.09	0.23	0.10
ζ_f	beta	0.25	0.1	0.22	0.09	0.23	0.10
ζ_f^*	beta	0.25	0.1	0.11	0.05	0.11	0.05
χ	beta	0.01	0.005	0.01	0.01	0.01	0.01
σ	beta	2	0.25	2.15	0.23	2.22	0.23
σ^*	beta	2	0.25	1.96	0.24	2.00	0.24
γ_r	beta	0.7	0.1	0.91	0.01	0.90	0.01
γ_π	norm	1.25	0.1	1.33	0.09	1.34	0.09
γ_y	beta	0.125	0.05	0.16	0.05	0.17	0.05
γ_r^*	beta	0.7	0.1	0.96	0.01	0.92	0.01
γ_π^*	norm	1.25	0.1	1.23	0.10	1.23	0.10
γ_y^*	beta	0.125	0.05	0.13	0.05	0.13	0.05

Table 4: Prior and posterior distribution: shocks

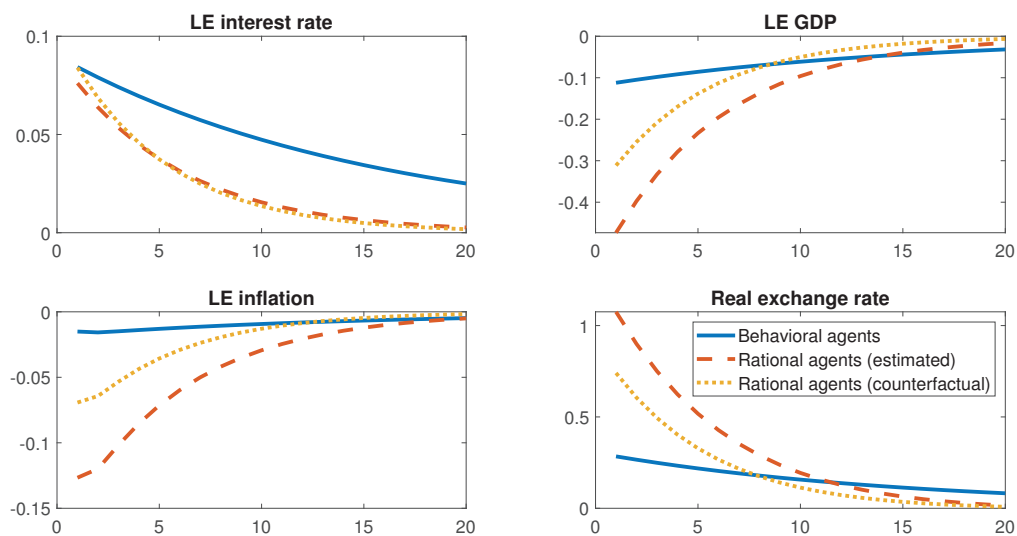
Parameter	Prior distribution			BE posterior distribution		RE posterior distribution	
	type	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
ρ_p	beta	0.7	0.1	0.72	0.08	0.70	0.08
ρ_p^*	beta	0.7	0.1	0.47	0.07	0.53	0.07
ρ_κ	beta	0.7	0.1	0.79	0.05	0.85	0.05
ρ_c	beta	0.7	0.1	0.70	0.07	0.88	0.07
ρ_c^*	beta	0.7	0.1	0.58	0.07	0.69	0.07
σ_ϕ	invg	0.003	Inf	0.0014	0.0001	0.0015	0.0001
σ_ϕ^*	invg	0.003	Inf	0.0009	0.0001	0.0009	0.0001
σ_p	invg	0.003	Inf	0.23	0.06	0.15	0.05
σ_p^*	invg	0.003	Inf	0.26	0.08	0.24	0.09
σ_κ	invg	0.01	Inf	0.016	0.0021	0.079	0.0016
σ_c	invg	0.01	Inf	0.037	0.0046	0.055	0.0064
σ_c^*	invg	0.01	Inf	0.037	0.0050	0.045	0.0053

Figure 1: Impulse responses to a contractionary monetary policy shock (small economy)



Note: Interest rate reactions in percentage points. Remaining variables in percent

Figure 2: Impulse responses to a contractionary monetary policy shock (large economy)



Note: Interest rate reactions in percentage points. Remaining variables in percent

Figure 3: Efficient monetary policy frontier in the small economy

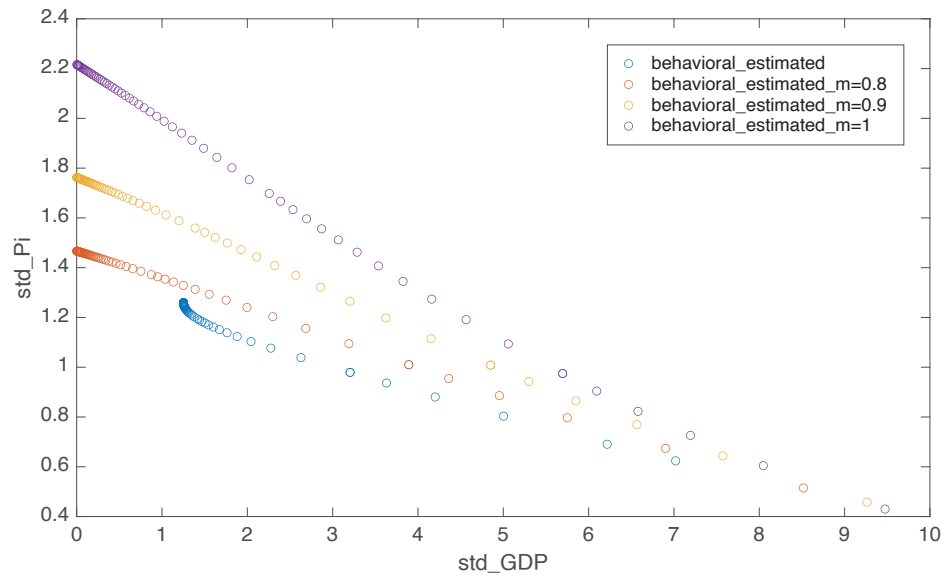


Figure 4: Efficient monetary policy frontier in the large economy

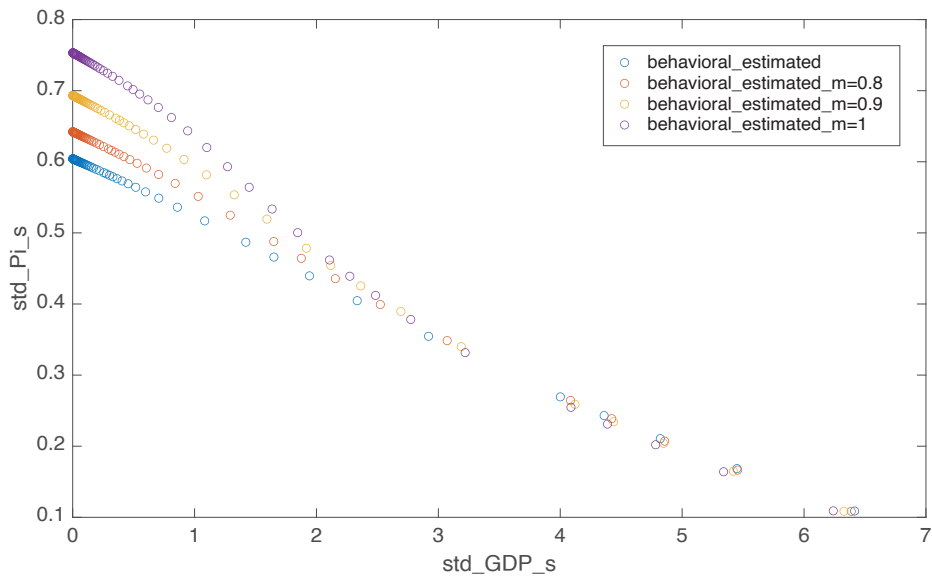
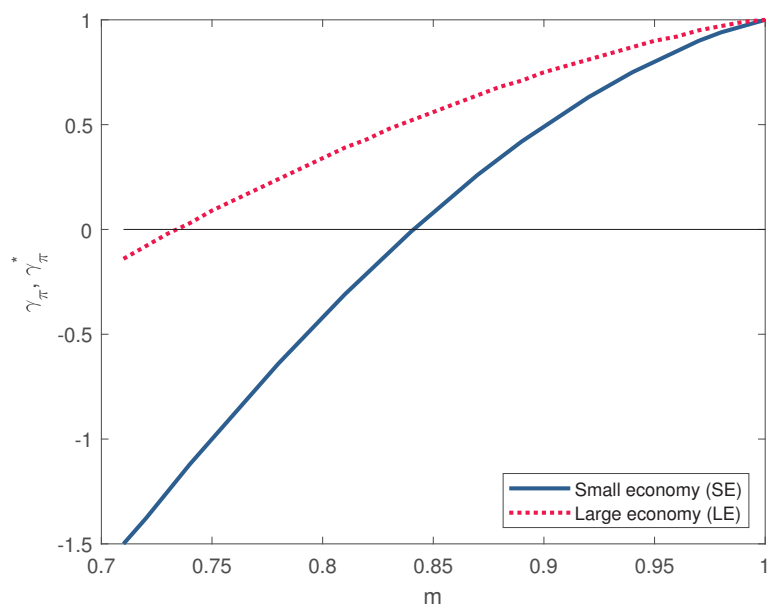
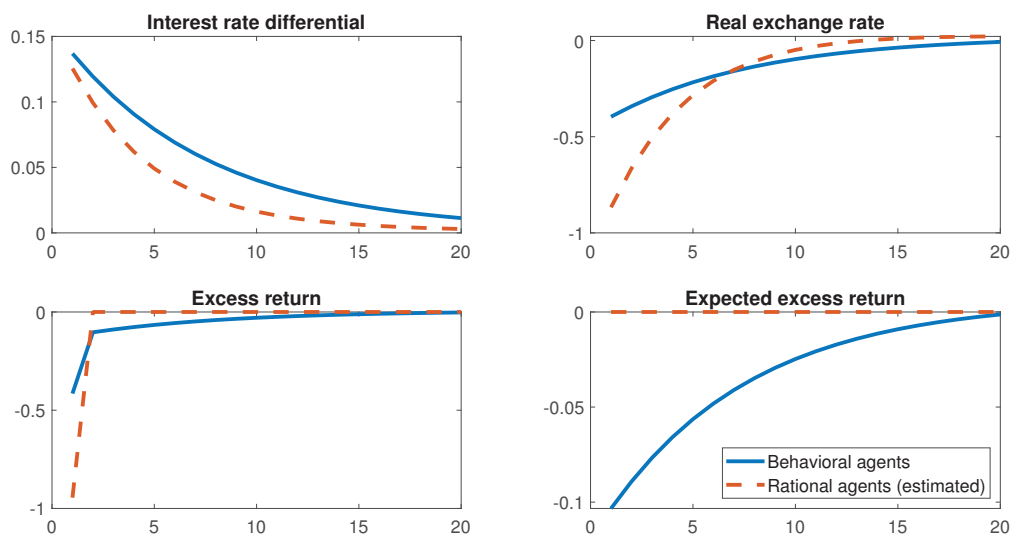


Figure 5: Determinacy boundaries for different levels of cognitive discounting



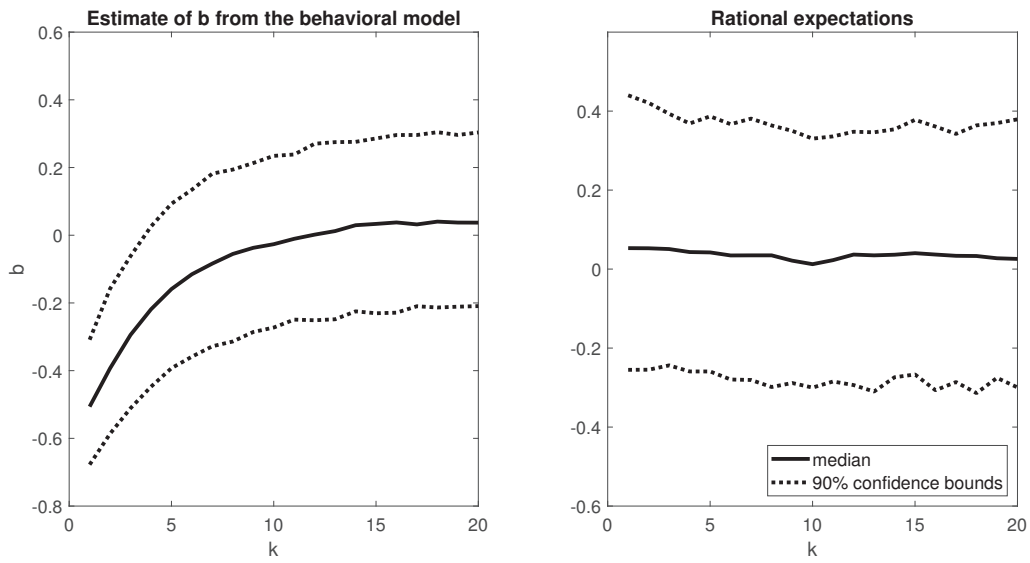
Note: The lines show the lowest values of γ_π and γ_π^* for which the Blanchard-Kahn condition remains satisfied. Blue solid line: small economy, red dotted line: large economy

Figure 6: UIP puzzle - impulse responses



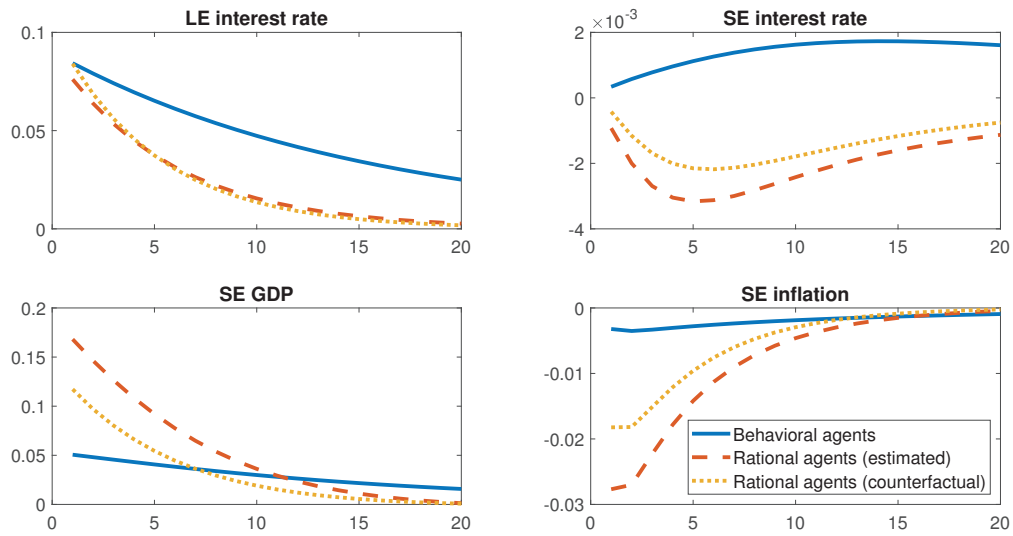
Note: The figure shows impulse responses to a monetary policy shock in the small economy. Interest rate reactions in percentage points. Remaining variables in percent

Figure 7: UIP puzzle - estimated coefficient b



Note: The figure reports estimated values for coefficient b from equation (26) for different values of k together with 90% confidence bounds.

Figure 8: Monetary policy independence in the SE



Note: The figure shows impulse responses to a contractionary monetary policy shock in the large economy. Interest rate reactions in percentage points. Remaining variables in percent

Appendix A

A.1 Derivation of the Euler and UIP equations

Households maximize their utility (1) subject to the budget constraint (3). While choosing consumption in period t , c_t they also 'choose' future consumption for all future periods and all possible states of the world in the future, $c_{t,t+k}$. This optimization yields the following first order conditions. First, we show a formula for intratemporal consumption-labor choice

$$\sigma \hat{c}_t = \hat{w}_t - \gamma \hat{n}_t$$

Next, we show equations obtained from differentiation with respect to domestic and foreign assets, respectively

$$\hat{c}_t = E_t^{BR} \hat{c}_{t,t+1} - \frac{1}{\sigma} E_t^{BR} (\hat{R}_t - \hat{\pi}_{t+1}) + \frac{1}{\sigma} (\hat{\varepsilon}_{u,t} - E_t^{BR} \hat{\varepsilon}_{u,t+1}) \quad (\text{A.1})$$

$$\hat{c}_t = E_t^{BR} \hat{c}_{t,t+1} - \frac{1}{\sigma} E_t^{BR} (\hat{R}_t^* + \hat{\kappa}_t - \hat{\pi}_{t+1}^*) - \frac{1}{\sigma} (E_t^{BR} \hat{q}_{t+1} - \hat{q}_t) + \frac{1}{\sigma} (\hat{\varepsilon}_{u,t} - E_t^{BR} \hat{\varepsilon}_{u,t+1}) \quad (\text{A.2})$$

where $E_t^{BR} \hat{c}_{t,t+1}$ denotes expected (over all possible states of the world tomorrow) consumption in period $t+1$ 'chosen' in period t . It is obtained from the problem with behavioral expectations about future variables (including incomes). Actual consumption in period $t+1$, c_{t+1} can be different (even ignoring the occurrence of shocks), because the consumer's decisions in period $t+1$ are based on expectations formed in period $t+1$.

UIP. Combining (A.1) and (A.2) yields the modified uncovered interest parity (UIP) condition

$$\hat{R}_t - m E_t \hat{\pi}_{t+1} = \hat{R}_t^* + \hat{\kappa}_t - m E_t \hat{\pi}_{t+1}^* + m \hat{q}_{t+1} - \hat{q}_t$$

Linearized budget constraint. Linearizing the budget constraint (3) yields

$$\begin{aligned} \bar{c} \hat{c}_t + \hat{b}_{H,t} + \bar{b}_H^* \hat{q}_t + \hat{b}_{H,t}^* &= \frac{1}{\mu} (\hat{w}_t + \hat{n}_t) + \bar{\Gamma} \hat{\Gamma}_t - \bar{T} \hat{T}_t \\ &+ \frac{1}{\beta} \bar{b}_H (\hat{R}_{t-1} - \hat{\pi}_t) + \frac{1}{\beta} \hat{b}_{H,t-1} + \frac{1}{\beta} \bar{b}_H^* (\hat{R}_{t-1}^* + \hat{\kappa}_{t-1} - \hat{\pi}_t^* + \hat{q}_t) + \frac{1}{\beta} \hat{b}_{H,t-1}^* \end{aligned} \quad (\text{A.3})$$

where $1/\mu = \bar{w}\bar{n}$ and $\bar{R}/\bar{\pi} = \bar{R}^*/\bar{\pi}^* = 1/\beta$.

Euler. Aggregating (A.3) yields

$$\begin{aligned}
 \left(\frac{\bar{c}}{\sigma} + \frac{1}{\mu\gamma}\right)\sigma\hat{c}_t &= (1-\beta)\left\{\frac{1}{\beta}\bar{b}_H(\hat{R}_{t-1} - \hat{\pi}_t) + \frac{1}{\beta}(\hat{b}_{H,t-1} + \hat{b}_{H,t-1}^*) + \frac{1}{\beta}\bar{q}\bar{b}_H^*(\hat{R}_{t-1}^* + \hat{\kappa}_{t-1} - \hat{\pi}_t^* + \hat{q}_t)\right. \\
 &+ (1-\beta)\left[\sum_{\tau=0}^{\infty}(m\beta)^\tau\left(\frac{1}{\mu}\frac{\gamma+1}{\gamma}E_t\hat{w}_{t+\tau} + \bar{\Gamma}E_t\hat{\Gamma}_{t+\tau} - \bar{T}E_t\hat{T}_{t+\tau}\right)\right. \\
 &- (1-\beta)\left(\frac{\bar{c}}{\sigma} - \frac{1-\beta}{\beta}(\bar{b}_H + \bar{q}\bar{b}_H^*) + \frac{1}{\mu\gamma}\right)\left[\sum_{\tau=1}^{\infty}\beta^\tau\left(\sum_{i=1}^{\tau}(m^{i-1}E_t\hat{R}_{t-1+i} - m^iE_t\hat{\pi}_{t+i})\right)\right. \\
 &\left.\left. + (1-\beta)\left(\frac{\bar{c}}{\sigma} + \frac{1}{\mu\gamma}\right)\left[\sum_{\tau=1}^{\infty}\beta^\tau(\hat{\varepsilon}_{u,t} - m^\tau E_t\hat{\varepsilon}_{u,t+\tau})\right]\right]\right\} \quad (\text{A.4})
 \end{aligned}$$

Next, after some algebra we can express the above equation (A.4) recursively

$$\begin{aligned}
 \hat{c}_t &= mE_t\hat{c}_{t+1} - \frac{1}{\sigma}(\hat{R}_t - mE_t\hat{\pi}_{t+1}) + \frac{1}{\sigma}(\hat{\varepsilon}_{u,t} - mE_t\hat{\varepsilon}_{u,t+1}) \\
 &+ \frac{1}{\left(\bar{c} + \frac{1}{\mu\gamma}\right)}\frac{(1-\beta)}{\beta}(1-m)(\hat{b}_{H,t} + \hat{b}_{H,t}^* + \bar{b}_H^*\hat{\kappa}_t)
 \end{aligned}$$

where $\bar{c} = 1 + \frac{1-\beta}{\beta}\bar{q}\bar{b}_H^* - \bar{g}$.

A.2 Derivation of the Phillips curve

Here we present derivation of the Phillips curve for home country production for the domestic market. All four Phillips curves are obtained in a similar fashion. Maximizing the intermediate good producer's profit (12) subject to the demand function (13) yields the following formula for the real price of domestic variety set in period t , $\hat{p}_{H,t}$

$$\hat{p}_{H,t} = (1 - \beta\theta_H)E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s [m^s(\hat{m}c_{t+s} + \hat{\varepsilon}_{p,t+s}) + \sum_{\nu=1}^s (m^\nu\hat{\pi}_{t+\nu} - m^{\nu-1}\zeta_h\hat{\pi}_{t-1+\nu})] \quad (\text{A.5})$$

and after some algebra can be expressed recursively

$$\hat{p}_{H,t} = (1 - \beta\theta_H)(\hat{m}c_t + \hat{\varepsilon}_{p,t}) + \beta\theta_H \left[mE_t\hat{p}_{H,t+1} + mE_t\hat{\pi}_{t+1} - \zeta_h\hat{\pi}_t \right]. \quad (\text{A.6})$$

Note that in this step, following Benchimol and Bounader (2021), we deviate from Gabaix (2020) while deriving equation (A.6). Next, from the zero profit condition of the final good producer we get

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{\frac{-1}{\mu}} di \right]^{-\mu} \quad (\text{A.7})$$

which after linearization and some rearranging yields

$$\hat{p}_{H,t} = \frac{1}{1-\theta_H} \hat{p}_{H,t} - \frac{\theta_H}{1-\theta_H} (\hat{p}_{H,t-1} + \zeta_h \hat{\pi}_{t-1} - \hat{\pi}_t) \quad (\text{A.8})$$

Substituting for \hat{p}_H from (A.8) into (A.6) after some algebra results in the following Phillips curve

$$\begin{aligned} \theta_H (\hat{p}_{H,t} - \hat{p}_{H,t-1} + \hat{\pi}_t - \zeta_H \hat{\pi}_{t-1}) = \\ (1-\theta_H)(1-\beta\theta_H)(\hat{m}c_t + \hat{\varepsilon}_{p,t} - \hat{p}_{H,t}) - (1-\theta_H)\beta\theta_H(1-m)(\hat{p}_{H,t} - \zeta_h \hat{\pi}_t) \\ + \beta\theta_H m E_t (E_t \hat{p}_{H,t+1} - \hat{p}_{H,t} + E_t \hat{\pi}_{t+1} - \zeta_H \hat{\pi}_t) \end{aligned} \quad (\text{A.9})$$

A.3 Model equations

A.3.1 Households

Euler equation. From (1), (2), (3) we obtain

$$\begin{aligned} \hat{c}_t = m E_t \hat{c}_{t+1} - \frac{1}{\sigma} (\hat{R}_t - m E_t \hat{\pi}_{t+1}) + \frac{1}{\sigma} (1 - m \rho_u) \hat{\varepsilon}_{u,t} \\ + \frac{1}{(\bar{c} + \frac{1}{\mu} \frac{\sigma}{\gamma})} \frac{(1-\beta)}{\beta} (1-m) (\hat{b}_{H,t} + \hat{b}_{H,t}^* + \bar{b}_H^* \hat{\kappa}_t) \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} \hat{c}_t^* = m E_t \hat{c}_{t+1}^* - \frac{1}{\sigma^*} (\hat{R}_t^* - m E_t \hat{\pi}_{t+1}^*) + \frac{1}{\sigma^*} (1 - m \rho_u^*) \hat{\varepsilon}_{u,t}^* \\ + \frac{1}{(\bar{c}^* + \frac{1}{\mu} \frac{\sigma^*}{\gamma^*})} \frac{(1-\beta^*)}{\beta^*} (1-m) (\hat{b}_{F,t}^* - \frac{\omega}{1-\omega} \hat{b}_{H,t}^*) \end{aligned} \quad (\text{A.11})$$

UIP condition.

$$\hat{R}_t - \hat{R}_t^* = m E_t \hat{q}_{t+1} - \hat{q}_t + m E_t \hat{\pi}_{t+1} - m \hat{\pi}_{t+1}^* + \hat{\kappa}_t \quad (\text{A.12})$$

Labor supply.

$$\sigma \hat{c}_t + \gamma \hat{l}_t = \hat{w}_t \quad (\text{A.13})$$

$$\sigma^* \hat{c}_t^* + \gamma^* \hat{l}_t^* = \hat{w}_t^* \quad (\text{A.14})$$

A.3.2 Producers

Denote $p_{H,t} = \frac{P_{H,t}}{P_t}$, $p_{F,t} = \frac{P_{F,t}}{P_t}$, $p_{H,t}^* = \frac{P_{H,t}^*}{P_t^*}$, $p_{F,t}^* = \frac{P_{F,t}^*}{P_t^*}$.

Demand for homogeneous intermediate goods. From (9) and (10) we obtain

$$\hat{y}_{H,t} = -\frac{1+\mu}{\mu}\hat{p}_{H,t} + \hat{y}_t \quad (\text{A.15})$$

$$\hat{y}_{F,t} = -\frac{1+\mu}{\mu}\hat{p}_{F,t} + \hat{y}_t \quad (\text{A.16})$$

$$\hat{y}_{H,t}^* = -\frac{1+\mu^*}{\mu^*}\hat{p}_{H,t}^* + \hat{y}_t^* \quad (\text{A.17})$$

$$\hat{y}_{F,t}^* = -\frac{1+\mu^*}{\mu^*}\hat{p}_{F,t}^* + \hat{y}_t^* \quad (\text{A.18})$$

The inflation of intermediate goods prices. From the definition of relative price $p_{d,t} = \frac{P_{d,t}}{P_t}$ and inflation of sector d intermediate good prices $\pi_{d,t} = \frac{P_{d,t}}{P_{d,t-1}}$, $d \in \{H, F\}$ we obtain.

$$\hat{\pi}_{H,t} = \hat{\pi}_t + \hat{p}_{H,t} - \hat{p}_{H,t-1} \quad (\text{A.19})$$

$$\hat{\pi}_{F,t} = \hat{\pi}_t + \hat{p}_{F,t} - \hat{p}_{F,t-1} \quad (\text{A.20})$$

and

$$\hat{\pi}_{F,t}^* = \hat{\pi}_t^* + \hat{p}_{F,t}^* - \hat{p}_{F,t-1}^* \quad (\text{A.21})$$

$$\hat{\pi}_{H,t}^* = \hat{\pi}_t^* + \hat{p}_{H,t}^* - \hat{p}_{H,t-1}^* \quad (\text{A.22})$$

Inflation of consumption good prices.

$$\hat{\pi}_t = \eta(\bar{p}_H)^{\frac{-1}{\mu}}(\hat{\pi}_{H,t} + \hat{p}_{H,t-1}) + (1-\eta)(\bar{p}_F)^{\frac{-1}{\mu}}(\hat{\pi}_{F,t} + \hat{p}_{F,t-1}) \quad (\text{A.23})$$

$$\hat{\pi}_t^* = \eta^*(\bar{p}_F^*)^{\frac{-1}{\mu}}(\hat{\pi}_{F,t}^* + \hat{p}_{F,t-1}^*) + (1-\eta^*)(\bar{p}_H^*)^{\frac{-1}{\mu}}(\hat{\pi}_{H,t}^* + \hat{p}_{H,t-1}^*) \quad (\text{A.24})$$

Marginal costs of intermediate goods.

$$\hat{m}c_t = \hat{w}_t \quad (\text{A.25})$$

$$\hat{m}c_t^* = \hat{w}_t^* \quad (\text{A.26})$$

Prices of intermediate goods. Prices of home goods follow from (12)

$$\begin{aligned} \theta_H(\hat{p}_{H,t} - \hat{p}_{H,t-1} + \hat{\pi}_t - \zeta_H\hat{\pi}_{t-1}) &= (1-\theta_H)(1-\beta\theta_H)(\hat{m}c_t - \hat{p}_{H,t} + \hat{\varepsilon}_{p,t}) \\ &\quad - (1-\theta_H)\beta\theta_H(1-m)\hat{p}_{H,t} \\ &\quad + \beta\theta_H E_t[m\hat{p}_{H,t+1} - m\hat{p}_{H,t} + M_{f,H}(\hat{\pi}_{t+1} - \zeta_H\hat{\pi}_t)] \end{aligned} \quad (\text{A.27})$$

and

$$\begin{aligned}\theta_H^*(\hat{p}_{H,t}^* - \hat{p}_{H,t-1}^* + \hat{\pi}_t^* - \zeta_H^* \hat{\pi}_{t-1}^*) &= (1 - \theta_H^*)(1 - \beta\theta_H^*)(\hat{m}c_t - \hat{q}_t - \hat{p}_{H,t}^* + \hat{\varepsilon}_{p,t}^*) \\ &\quad - (1 - \theta_H^*)\beta\theta_H^*(1 - m)\hat{p}_{H,t}^* \\ &\quad + \beta\theta_H^*E_t[m\hat{p}_{H,t+1}^* - m\hat{p}_{H,t}^* + M_{f,H}^*(\hat{\pi}_{t+1}^* - \zeta_H^* \hat{\pi}_t^*)]\end{aligned}\quad (\text{A.28})$$

where

$$\begin{aligned}M_{f,H} &= m\theta_H + m(1 - \theta_H)\frac{1 - \beta\theta_H}{1 - \beta\theta_H m} \\ M_{f,H}^* &= m\theta_H^* + m(1 - \theta_H^*)\frac{1 - \beta\theta_H^*}{1 - \beta\theta_H^* m}\end{aligned}$$

Prices of foreign goods

$$\begin{aligned}\theta_F^*(\hat{p}_{F,t}^* - \hat{p}_{F,t-1}^* + \hat{\pi}_t^* - \zeta_F^* \hat{\pi}_{t-1}^*) &= (1 - \theta_F^*)(1 - \beta^*\theta_F^*)(\hat{m}c_t^* - \hat{p}_{F,t}^* + \hat{\varepsilon}_{p,t}^*) \\ &\quad - (1 - \theta_F^*)\beta^*\theta_F^*(1 - m)\hat{p}_{F,t}^* \\ &\quad + \beta^*\theta_F^*E_t[m\hat{p}_{F,t+1}^* - m\hat{p}_{F,t}^* + M_{f,F}^*(\hat{\pi}_{t+1}^* - \zeta_F^* \hat{\pi}_t^*)]\end{aligned}\quad (\text{A.29})$$

and

$$\begin{aligned}\theta_F(\hat{p}_{F,t} - \hat{p}_{F,t-1} + \hat{\pi}_t - \zeta_F \hat{\pi}_{t-1}) &= (1 - \theta_F)(1 - \beta^*\theta_F)(\hat{m}c_t^* + \hat{q}_t - \hat{p}_{F,t} + \hat{\varepsilon}_{p,t}) \\ &\quad - (1 - \theta_F)\beta^*\theta_F(1 - m)\hat{p}_{F,t} \\ &\quad + \beta^*\theta_F E_t[m\hat{p}_{F,t+1} - m\hat{p}_{F,t} + M_{f,F}(\hat{\pi}_{t+1}^* - \zeta_F \hat{\pi}_t^*)]\end{aligned}\quad (\text{A.30})$$

where

$$\begin{aligned}M_{f,F}^* &= m\theta_F^* + m(1 - \theta_F^*)\frac{1 - \beta^*\theta_F^*}{1 - \beta^*\theta_F^* m} \\ M_{f,F} &= m\theta_F + m(1 - \theta_F)\frac{1 - \beta^*\theta_F}{1 - \beta^*\theta_F m}\end{aligned}$$

Aggregate production function. Home industry:

$$\bar{y}_H \hat{y}_{H,t} + (1 - \bar{y}_H) \hat{y}_{H,t}^* = \hat{l}_t \quad (\text{A.31})$$

and foreign industry

$$\bar{y}_F^* \hat{y}_{F,t} + (1 - \bar{y}_F^*) \hat{y}_{F,t}^* = \hat{l}_t^* \quad (\text{A.32})$$

A.3.3 Government

Government expenditures and taxes.

$$\frac{1}{\bar{y}}\hat{b}_{G,t} + \frac{\bar{T}}{\bar{y}}\hat{T}_t = \frac{\bar{b}_G}{\bar{y}}\hat{r}_t + \frac{1}{\bar{y}\beta}\hat{b}_{G,t-1} + \frac{\bar{g}}{\bar{y}}\hat{g}_t \quad (\text{A.33})$$

$$\frac{1}{\bar{y}^*}\hat{b}_{G,t}^* + \frac{\bar{T}^*}{\bar{y}^*}\hat{T}_t^* = \frac{\bar{b}_G^*}{\bar{y}^*}\hat{r}_t^* + \frac{1}{\bar{y}^*\beta^*}\hat{b}_{G,t-1}^* + \frac{\bar{g}^*}{\bar{y}^*}\hat{g}_t^* \quad (\text{A.34})$$

$$\bar{T}\hat{T}_t = \nu\hat{b}_{G,t} \quad (\text{A.35})$$

$$\bar{T}^*\hat{T}_t^* = \nu^*\hat{b}_{G,t}^* \quad (\text{A.36})$$

A.3.4 Central Bank

Taylor rule. From (19) we obtain

$$\hat{R}_t = \gamma_R\hat{R}_{t-1} + (1 - \gamma_R)(\gamma_\pi\hat{\pi}_t + \gamma_y\hat{y}_t) + \varphi_t \quad (\text{A.37})$$

$$\hat{R}_t^* = \gamma_R^*\hat{R}_{t-1}^* + (1 - \gamma_R^*)(\gamma_\pi^*\hat{\pi}_t^* + \gamma_y^*\hat{y}_t^*) + \varphi_t^* \quad (\text{A.38})$$

A.3.5 Closing the model

Goods market.

$$\frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{g}}{\bar{y}}\hat{g}_t = \hat{y}_t \quad (\text{A.39})$$

$$\frac{\bar{c}^*}{\bar{y}^*}\hat{c}_t^* + \frac{\bar{g}^*}{\bar{y}^*}\hat{g}_t^* = \hat{y}_t^* \quad (\text{A.40})$$

Balance of Payments. From (21) we obtain

$$\begin{aligned} \frac{\bar{q}\bar{p}_{H,t}^*\bar{y}_{H,t}^*}{\bar{y}}\frac{1-\omega}{\omega}(\hat{q}_t + \hat{p}_{H,t}^* + \hat{y}_{H,t}^*) - \frac{1}{\bar{y}}\hat{b}_{H,t}^* = \\ \left(\frac{\bar{q}\bar{p}_{H,t}^*\bar{y}_{H,t}^*}{\bar{y}}\frac{1-\omega}{\omega} - \frac{1-\beta}{\beta}\frac{\bar{b}_H}{\bar{y}}\right)(\hat{p}_{F,t} + \hat{y}_{F,t}) \\ + \frac{\bar{b}_H}{\bar{y}\beta^*}(\hat{q}_t - \hat{q}_{t-1} + \hat{R}_{t-1}^* + \kappa_{t-1} - \hat{\pi}_t^*) - \frac{1}{\bar{y}\beta}\hat{b}_{H,t-1}^* \end{aligned} \quad (\text{A.41})$$

Risk premium. From (2) we obtain

$$\hat{\kappa}_t = -\chi\left(\frac{\bar{b}_s}{\bar{y}} + \bar{b}_H\hat{y}_t\right) + \hat{\varepsilon}_{\kappa,t} \quad (\text{A.42})$$

GDP. From (20) we obtain

$$\hat{y}_t = \left(1 - \frac{1 - \beta}{\beta} \bar{b}_H\right) \hat{y}_t + \frac{\bar{q} \bar{p}_{H,t}^* \bar{y}_{H,t}^*}{\bar{y}} \frac{1 - \omega}{\omega} (\hat{p}_H + \hat{y}_H + \hat{q}_t) - \left(\frac{\bar{q} \bar{p}_{H,t}^* \bar{y}_{H,t}^*}{\bar{y}} \frac{1 - \omega}{\omega} - \frac{1 - \beta}{\beta} \frac{\bar{b}_H}{\bar{y}}\right) (\hat{p}_F + \hat{y}_F) \quad (\text{A.43})$$

$$\hat{y}_t^* = \left(1 - \frac{1 - \beta^*}{\beta^*} \bar{b}_H^*\right) \hat{y}_t^* + \left(\frac{\bar{q} \bar{p}_{H,t}^* \bar{y}_{H,t}^*}{\bar{y}^*} + \frac{1 - \beta^*}{\beta^* \bar{y}^*} \bar{b}_H^*\right) (\hat{p}_F + \hat{y}_F - \hat{q}_t) - \frac{\bar{q} \bar{p}_{H,t}^* \bar{y}_{H,t}^*}{\bar{y}^*} (\hat{p}_H + \hat{y}_H) \quad (\text{A.44})$$

where $\bar{b}_H^* = -\frac{\omega}{1 - \omega} \bar{b}_H$.

www.nbp.pl

