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## The Macroeconomic Effects of Losing Autonomous Monetary Policy after the Euro Adoption in Poland

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### Abstract

There are many issues associated with the Eurozone accession of Poland. The goal of this paper is to analyse one, but very important aspect, namely - the macroeconomic impact of the loss of autonomous monetary policy. In order to answer this question, we build a two country DSGE model with sticky prices. We begin by evaluating the performance of our model. Next, we investigate how joining the Eurozone will affect the business cycle behaviour of the main macroeconomic variables in Poland. We find that the Euro adoption will have a noticeable impact on the Polish economic fluctuations. In particular, the volatility of domestic output increases and the volatility of inflation decreases. Also, in order to quantify the effect of the Euro adoption, we compute the welfare effect of this monetary policy change. Our findings suggest that the welfare cost is not large.

*JEL:* E52, E58, F41, F33.

*Keywords:* Monetary policy, EMU accession countries, Euro.

# 1 Introduction

With the accession to the European Union in 2004, Poland, as well as the other New Member States, agreed to adopt the Euro as the national currency and become a member of the Euro area. However, the accession to the Euro area implies important changes in both the conduct of macroeconomic policy and the behaviour of the accessing economy. The most important of these are:

- fixing of the exchange rate against the other participants of the Euro area,
- resignation from the autonomous monetary policy in favour of the common monetary policy conducted by the ECB.

These changes bring both benefits and costs for the accessing country.

To a large extent, the existing literature focused on welfare benefits associated with the membership in the monetary union. Rose (2000) and Frankel and Rose (2002) argued that the accession to the monetary union boosts trade, creating welfare gains. But their estimate of the magnitude of this effect met some criticism - see e.g. Faruqee (2004). In case of the Polish economy, the important contribution of Daras and Hagemeyer (2008) shows the benefits of the Euro adoption, taking into account the trade creation effect and a decline of the long-term real interest rate through elimination of the risk premium. Their results were calculated using dynamic Computable General Equilibrium (CGE) framework.

Our contribution to the discussion on the consequences of the Euro adoption concentrates on the costs associated with abandoning autonomous monetary policy. So, after the accession to the Euro area, monetary policy will be conducted by the European Central Bank and will be responding, in the first place, to the events affecting the whole Euro area. In the presence of the asymmetric shocks, affecting mainly a given member country, common monetary policy will be less suited to the current situation of this country, giving rise to the additional volatility of the economic development and the associated welfare costs of these fluctuations.

Thus, abstracting from other aspects of monetary integration, our analysis aims only at assessing the impact of the resignation from autonomous monetary policy on the volatility of the economic development, measured by a set of main macroeconomic indicators. We are also going to provide an estimate of the welfare costs associated with the monetary policy change.

In order to perform this calculation, we propose a Dynamic Stochastic General Equilibrium model (DSGE<sup>1</sup>) of two economies (Poland and the

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<sup>1</sup>This framework builds on the seminal paper of Kydland and Prescott (1982) and the Real Business Cycle school, which was enhanced with Keynesian-type of nominal rigidities, like in the important work of Smets and Wouters (2003), resulting in neoclassical synthesis - see e.g. Goodfriend and King (1998).

Euro area) linked through trade in goods and services and incomplete international assets markets. In order to specify properly the parameters of the model, we decided to estimate a relatively large number of parameters using the data from both economies. It allows us to develop a model that mimics closely the behaviour of both economies.

The DSGE methodology is relatively well suited to analyse the consequences of the monetary policy regime switch. The model describes the laws of motion of the economy which are derived from microeconomic foundations. In other words, all agents populating the economy solve well specified decision problems and respond in an optimal way to changes in economic environment. As the economy is described in terms of preferences, technologies and the rules of market clearing, the model is parametrized in terms of so called “deep” parameters. It implies that these parameters are invariant to changes in policies and environments and it is possible to analyse the consequences of these changes in a way that is immune to the Lucas critique (see Lucas, 1976). Additionally, agents populating the economy, while making their decisions, form expectations (under the assumption of rationality) about the future, so the model incorporates expectations in an internally coherent way. All these features make the DSGE framework a parsimonious tool to analyse the consequences of the resignation from autonomous monetary policy in Poland.

According to the standard economic theory, monetary policy is neutral in the long run, so the change of the monetary policy regime should not affect the economy in the long run. Thus, the monetary policy regime switch should influence the volatility, rather than the level or the growth rate of the economy. As households are usually assumed to be risk averse, they dislike high variation in income and consumption, thus higher volatility of the economic growth generates the welfare costs for households. Since households preferences are directly specified in the DSGE framework, we are able to assess the consequences of Euro adoption not only in terms of the volatility of macroeconomics variables (as in Karam, Laxton, Rose, and Tamirisa, 2008), but also in terms of the consumer welfare (as in Lucas, 1987).

Our approach is directly related to the literature on the costs of business cycle fluctuations, which starts from the seminal contribution of Lucas (1987). Lucas finds that the costs of economic fluctuations are quite small - his estimate for the US economy implies that individuals would sacrifice at most 0.1% of their lifetime consumption (his point estimate under reasonable calibration of the prototype economy amounts to 0.008%). The result of Lucas was quite controversial and launched subsequent research. Barlevy (2004a) reviews the literature on this topic and finds that the estimates of the costs of business cycles range from 0.01% to 2% of lifetime consumption. The higher estimates are usually obtained with either non-standard preferences (e.g. Epstein-Zin) or high risk aversion (calibrated to match household micro data, which is inconsistent with the macro evidence for the class of CRRA preferences).

The standard models of the business cycle fluctuations generate relatively low estimates of the business cycle costs. On the one hand, when thinking about fluctuations one usually thinks of recessions, as times when people are worse off, but the lack of economic fluctuations also means that there are no economic expansions (when individuals are actually better off). On the other hand, since agents dislike economic fluctuations, they do their best to smooth them out. Additionally, the standard models of business cycle fluctuations imply that there are no long-run consequences of these, so there is no level effect of fluctuations (and thus the magnitude of the welfare losses is of the second order).

There is also literature which argues that there is a level effect. The so-called endogenous business cycle literature (see e.g. Barlevy, 2004b) assumes that there are long run effects of economic fluctuations, so fluctuations have the level effect on welfare, which results in much higher costs of cyclical fluctuations - an order of magnitude higher, e.g. 8% in Barlevy (2004b). This approach builds mainly on the empirical evidence showing that higher volatility of economic growth is usually associated with lower average growth rates. But the tools of this approach are mostly econometric, thus the causality still remains unsolved. As far as we know, there does not exist a general equilibrium model incorporating these relationships, which is well grounded in the data. Due to this shortcoming and the fact that the literature on the endogenous cycle is not in the mainstream of economic thinking, we choose the first approach to the business cycle.

The literature on the costs of the Euro adoption (or more broadly - monetary union) is rather limited. Ca' Zorzi, De Santis, and Zampolli (2005) argue that adopting the Euro is more likely to be welfare enhancing, the higher the relative volatility of supply shocks across the participating countries, the smaller the correlation of countries' supply shocks and the larger the variance of real exchange rate shocks. Additionally, the welfare effects do not depend on deterministic factors influencing the real exchange rate (such as Balassa-Samuelson effect), but rather on variances and covariances of shocks. They also claim, that the Euro area accession decreases the effectiveness of the monetary policy response to the stochastic shocks, so it creates a cost of monetary union. However, the Euro adoption could be beneficial if the positive impact on potential output (through the trade creation channel) is higher than the negative effect of lower monetary policy effectiveness.

Karam, Laxton, Rose, and Tamirisa (2008) use a DSGE model developed in the IMF to assess the implications of the Euro adoption on the volatility of an accessing country (the model is roughly calibrated to data from the Czech Republic - the authors argue that it is a typical New Member economy<sup>2</sup>). Their results show that the accession of a small country to the Euro

<sup>2</sup>This argument is very stylized as there are many features that distinguish Eastern European countries, especially in terms of volatility of GDP components - for some discussion, see Choueiri, Murgasova, and Székely (2005).



area increases the volatility of both inflation and output, as the exchange rate no longer buffers a part of the volatility generated by economic shocks<sup>3</sup>. The increase of the volatility of the economic fluctuations gets smaller with the fiercer competition in the goods market, the smaller rigidities, and the greater trade integration with the Euro area. The approach of Karam, Laxton, Rose, and Tamirisa (2008) provides many interesting answers, although it focuses on the volatility effects of Euro adoption, emphasizing the problem of the inflation-output volatility faced by the central bank. They do not try to quantify the welfare results of the Euro area accession.

Lopes (2007) uses a framework that is the most closely related to ours. She also uses a symmetric two-country DSGE model, with capital and nominal rigidities<sup>4</sup>, but she calibrates all of the model parameters. The latter is relatively hard in case of the New Member States as the research on technologies, preferences and market structures in these economies is rather scarce. She finds the welfare costs of losing monetary policy independence to be 0.25% of lifetime consumption (in case of Poland). We believe that her result might be biased since, as far as we understand, it is based on only one draw of random shocks for about 1000 periods. Our experiments show that one needs a large number of draws for the convergence of the welfare result. She also finds the level effects, which in our computations disappear as the number of draws increases.

The rest of the paper is organized as follows. In Section 2 we describe our basic model economy. Section 3 discusses the calibration and the estimation issues and shows the performance of the model, both in terms of impulse response functions and moments of the model variables against the data. In Section 4 we present and discuss the results of the model both in terms of the volatilities of economic aggregates, as well as in terms of the consumer welfare. Section 5 concludes the paper.

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<sup>3</sup>The authors argue that "...flexible exchange rate plays an important buffering role that facilitates macroeconomic adjustment to shocks in small, emerging economies, which allows the central bank to achieve better outcomes in terms of domestic volatility. In general, the results show that there is a cost to a small, emerging economy in joining a common currency area when this flexibility is lost. The essential reason is that there are rigidities in domestic adjustment, and when the burden of macroeconomic adjustment is forced onto domestic nominal variables under the common currency, macroeconomic volatility generally increases..." - Karam, Laxton, Rose, and Tamirisa (2008), page 354.

<sup>4</sup>Although she uses the staggered price setting in the spirit of Taylor (1980) and we choose a framework of Calvo (1983), that is more frequently used in the DSGE literature.

## 2 Model

We employ the standard Dynamic Stochastic General Equilibrium model of business cycle with nominal rigidities. In our model, monopolistic producers set prices in a style proposed by Calvo (1983). We build a two country model in the tradition of Chari, Kehoe, and McGrattan (2002). In the model we name Poland home and the Eurozone foreign. Monetary policy is modelled as an interest rate rule similar to the one proposed by Taylor (1993). Our model shares many features with closed economy models (including Erceg, Henderson, and Levin, 2000, Smets and Wouters, 2003), and small open macro economy models (including Altig, Christiano, Eichenbaum, and Lindé, 2005, Christiano, Eichenbaum, and Evans, 2005, Adolfson, Laséen, Lindé, and Villani, 2005), and other two country models (including Lopes, 2007 and Rabanal and Tuesta, 2007).

Households can save in domestic bonds and/or international bonds. We assume that domestic bonds markets are complete but international bonds markets are incomplete. Introduction of this market structure leads to the arbitrage condition that in a log-linearised version takes a form of the uncovered interest rate parity (UIP) condition. Households also decide how much capital to rent to producers (utilization rate) and choose how much to invest in new capital. Furthermore, households supply labour in the competitive labour market.

There are three stages of the production process. In the first stage producers offer differentiated products to both domestic and foreign second stage producers. They set their prices according to the Calvo scheme. By including the nominal rigidities in the buyer's currency, we obtain the incomplete exchange rate pass-through. In the remaining two stages, perfectly competitive producers combine those differentiated goods into a single consumption/investment good with domestic and foreign component.

Next, we describe in detail the optimization problems of consumers and producers as well as the behaviour of fiscal and monetary authorities.

### 2.1 Households

There is a continuum of households of measure 1. The fraction  $\omega$  of households reside in the home country and the fraction  $1 - \omega$  of households reside in the foreign country. Herein we describe the problem of the home country household. The problem of the foreign country household is defined analogously.

Households supply labour, consume goods, trade domestic bonds in the complete markets, and international bonds in the incomplete markets. They also choose the capital utilization rate and accumulate capital. The representative household's preferences are of the form <sup>5</sup>

<sup>5</sup>The convention employed in this paper is that asterisk denotes the counterpart in

$$W_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, \zeta_t) \right], \quad (1)$$

where  $c_t$  and  $l_t$  denote the representative household's consumption and labour supply, respectively.  $\zeta_t$  denotes the labour supply shock that follows AR(1) process

$$\hat{\zeta}_t = \rho_{\zeta} \hat{\zeta}_{t-1} + \varepsilon_{\zeta,t}, \quad (2)$$

where  $E_t[\zeta_t] = 1$  and  $\hat{\zeta}_t = \zeta_t - 1$  (in the deterministic steady state  $\bar{\zeta} = 1$ ). Throughout the whole paper we assume the following form of the instantaneous utility function

$$u(c_t, l_t, \zeta_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \psi \zeta_t \frac{l_t^{1+\gamma}}{1+\gamma}. \quad (3)$$

We restrict our analysis to the case of a representative consumer by assuming complete domestic financial markets. In period  $t$  there is a complete set of state contingent one-period nominal bonds  $B_{t+1}$ , each worth  $\Upsilon_{t,t+1}$ . Households can also trade bonds with abroad. We assume that there is one, internationally traded, uncontingent nominal bond, nominated in the foreign currency  $D_{t+1}^*$  (denote the home country holdings of this bond as  $D_{H,t+1}^*$ , and the foreign country holdings of this bond as  $D_{F,t+1}^*$ ). From the point of view of the home country households the interest rate on this bond is  $R_t^* \kappa_t$ , where  $\kappa_t$  denotes risk premium. The risk premium is a function of the domestic debt (as in Schmitt-Grohe and Uribe, 2003)

$$\kappa_t = \exp \left( -\chi \left( \frac{e_t D_{H,t+1}^*}{P_t \tilde{Y}_t} - d \right) \right) \epsilon_{\kappa,t}, \quad (4)$$

where  $e_t$ ,  $P_t$  and  $\tilde{Y}_t$  denote the nominal exchange rate, the price of the consumption good and GDP, respectively. The constant  $d$  is calibrated so that there is no risk premium in the deterministic steady state.  $\epsilon_{\kappa,t}$  denotes the risk premium shock that follows AR(1) process

$$\hat{\epsilon}_{\kappa,t} = \rho_{\kappa} \hat{\epsilon}_{\kappa,t-1} + \varepsilon_{\zeta,t}, \quad (5)$$

where  $E_t[\epsilon_{\kappa,t}] = 1$  and  $\hat{\epsilon}_{\kappa,t} = \epsilon_{\kappa,t} - 1$  (in the deterministic steady state  $\bar{\epsilon}_{\kappa} = 1$ ).

Moreover, households accumulate capital. The dynamics of the stock of capital follows the law of motion

$$\tilde{k}_{t+1} = (1 - \delta) \tilde{k}_t + \left( 1 - S \left( \frac{x_t}{x_{t-1}} \right) \right) x_t, \quad (6)$$

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the foreign country of a variable in the home country (for example  $c_t$  is consumption in the home country, and  $c_t^*$  is consumption in the foreign country). The same applies to the model's parameters. Whenever we see potential for confusion we explicitly clarify notation.

where  $\tilde{k}_t$  and  $x_t$  denote the capital stock and investment.  $S(x_t/x_{t+1})$  is a function which transforms investment into physical capital and introduces the capital adjustment costs into the model. We adopt the specification of Christiano, Eichenbaum, and Evans (2005) and assume that in the deterministic steady state there are no capital adjustment costs ( $S(1) = S'(1) = 0$ ), and the function is concave in the neighbourhood of the deterministic steady state ( $\iota = 1/S''(1) > 0$ ). Households also choose the capital utilization rate<sup>6</sup>  $u_t \in [0, \infty)$ , and have to pay the capital utilization rate adjustment costs  $\Psi(u_t)\tilde{k}_t$ , where  $\Psi(u_t)$  is increasing and convex. Furthermore, we assume no utilization adjustment cost in the deterministic steady state, so the function  $\Psi(u_t)$  satisfies  $\Psi(1) = 0$ ,  $\Psi'(1) > 0$  and  $\Psi''(1) > 0$ . Moreover, households rent capital to producers. Denote the capital stock employed by producers as  $k_t$ , then

$$k_t = u_t \tilde{k}_t \quad (7)$$

and the rental rate of capital is denoted as  $r_t$ .

All households in the home country face the same budget constraint in each period

$$c_t + x_t + \frac{E_t [\Upsilon_{t,t+1} B_{t+1}]}{P_t} + \frac{D_{H,t+1}^* e_t}{P_t R_t^* \kappa_t} = w_t l_t + (r_t u_t - \Psi(u_t)) \tilde{k}_t + \frac{B_t}{P_t} + \frac{e_t D_{H,t}^*}{P_t} - T_t + \Pi_t, \quad (8)$$

where  $P_t$ ,  $w_t$ ,  $T_t$  and  $\Pi_t$  denote the price of consumption good, the real wage rate, real lump sum tax and real profits from all producers, respectively.

The representative home country household maximizes (1) subject to the budget constraint (8) (denote the Lagrangian multiplier on budget constraint as  $\lambda_t$ ), the law of motion of capital (6) (denote the Lagrangian multiplier on budget constraint as  $\lambda_t Q_t$ ), and the standard no-Ponzi game condition.

Solving, we get the following first order conditions

$$c_t : \beta^t u_{c,t} = \lambda_t, \quad (9)$$

$$l_t : \beta^t u_{l,t} = -\lambda_t w_t, \quad (10)$$

$$x_t : \lambda_t = \lambda_t Q_t \left( 1 - S \left( \frac{x_t}{x_{t-1}} \right) - S' \left( \frac{x_t}{x_{t-1}} \right) \frac{x_t}{x_{t-1}} \right) + E_t \left[ Q_{t+1} \lambda_{t+1} \left( S' \left( \frac{x_{t+1}}{x_t} \right) \left( \frac{x_{t+1}}{x_t} \right)^2 \right) \right], \quad (11)$$

$$\tilde{k}_{t+1} : E_t [\lambda_{t+1} (r_{t+1} u_{t+1} - \Psi(u_{t+1}))] = Q_t \lambda_t - E_t [Q_{t+1} \lambda_{t+1} (1 - \delta)], \quad (12)$$

$$u_t : r_t = \Psi'(u_t), \quad (13)$$

<sup>6</sup> $u_t$  is normalized, so that 1 denotes the deterministic steady state capacity utilization rate.

$$B_{t+1} : E_t [\Upsilon_{t,t+1}] = E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \right], \quad (14)$$

$$D_{H,t+1}^* : \frac{1}{R_t^* \kappa_t} = E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \frac{e_{t+1}}{e_t} \right], \quad (15)$$

and the transversality conditions. Define the nominal interest rate in home country as

$$\frac{1}{R_t} = E_t [\Upsilon_{t,t+1}]. \quad (16)$$

The log-linearised equations can be found in Appendix A.

## 2.2 Producers

Herein we describe the actions of producers in the home country, producers in the foreign country act analogously. There are three stages of the production process in both economies. We describe the decision problems of the producers, moving from the top (final good producers), through the middle (homogeneous intermediate good producers), to the bottom (heterogeneous intermediate good producers) of the production process in the economy.

In the last stage, final domestic goods producers buy home and foreign homogeneous intermediate goods and combine them into domestic consumption/investment goods that are sold to consumers. In the second stage there are two sectors:  $H$  and  $F$ . In sector  $H$  producers buy home heterogeneous intermediate goods and aggregate them into home homogeneous intermediate goods that are sold to the domestic final goods producers. Similarly in sector  $F$ , producers buy foreign heterogeneous intermediate goods and aggregate them into foreign homogeneous intermediate goods that are sold to the domestic final goods producers. In the first stage, heterogeneous intermediate goods producers use capital and labour to produce heterogeneous intermediate goods that are sold both at home and at foreign. Next we describe the problems of producers in the home country in more detail. In terms of notation, goods produced at home are sub-scripted with an  $H$ , while those produced abroad are sub-scripted with an  $F$ . Moreover, allocations and prices in the foreign country are denoted with an asterisk.

### 2.2.1 Final Goods Producers

Final goods producers operate in a perfectly competitive market. They buy sector  $H$  homogeneous intermediate goods  $Y_{H,t}$  and sector  $F$  homogeneous intermediate goods  $Y_{F,t}$  at competitive prices  $P_{H,t}$  and  $P_{F,t}$ , respectively. The final goods producers use those goods to produce final goods  $Y_t$  using the following technology

$$Y_t = \left[ \eta^{\frac{\mu}{1+\mu}} (Y_{H,t})^{\frac{1}{1+\mu}} + (1 - \eta)^{\frac{\mu}{1+\mu}} (Y_{F,t})^{\frac{1}{1+\mu}} \right]^{1+\mu}, \quad (17)$$

which are sold to home country consumers. Since markets are competitive, the final good producers take prices as given and, in each period  $t$ , choose inputs and output to maximize profits given by

$$P_t Y_t - P_{H,t} Y_{H,t} - P_{F,t} Y_{F,t} \quad (18)$$

subject to the production function (17).

Solving the problem in (18) gives the inputs demand functions

$$Y_{H,t} = \eta \left( \frac{P_{H,t}}{P_t} \right)^{\frac{-(1+\mu)}{\mu}} Y_t, \quad (19)$$

$$Y_{F,t} = (1 - \eta) \left( \frac{P_{F,t}}{P_t} \right)^{\frac{-(1+\mu)}{\mu}} Y_t. \quad (20)$$

Using the zero profit condition, we construct the index of domestic prices

$$P_t = \left[ (1 - \eta) (P_{F,t})^{\frac{-1}{\mu}} + \eta (P_{H,t})^{\frac{-1}{\mu}} \right]^{-\mu}. \quad (21)$$

### 2.2.2 Homogeneous Intermediate Goods Producers

Home and foreign homogeneous intermediate goods producers operate in a competitive market. Home homogeneous intermediate goods producers buy a continuum of home heterogeneous intermediate goods  $y_H(i)$ ,  $i \in [0, 1]$ , while foreign homogeneous intermediate goods producers  $F$  buy a continuum of foreign heterogeneous intermediate goods  $y_F(i)$ ,  $i \in [0, 1]$ . Prices of both home and foreign heterogeneous intermediate goods are set in a currency of the home country, denoted as  $p_{H,t}(i)$  and  $p_{F,t}(i)$ , respectively. The technology for producing home homogeneous intermediate goods is as follows

$$Y_{H,t} = \left( \int_0^1 y_{H,t}(i)^{\frac{1}{1+\mu_H}} di \right)^{1+\mu_H}. \quad (22)$$

Given prices, in each period  $t$ , the home homogeneous intermediate goods producers choose inputs and output to maximize profits

$$P_{H,t} Y_{H,t} - \int_0^1 p_{H,t}(i) y_{H,t}(i) di \quad (23)$$

subject to the production function (22).

Solving the problem of the producers in (23) we obtain the following input demands for home heterogeneous intermediate goods

$$y_{H,t}(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{\frac{-(1+\mu_H)}{\mu_H}} Y_{H,t}. \quad (24)$$

From the zero profit condition we also get the price index

$$P_{H,t} = \left[ \int_0^1 p_{H,t}(i)^{\frac{-1}{\mu_H}} di \right]^{-\mu_H}. \quad (25)$$

The problem of the foreign homogeneous intermediate goods producers is analogous. Thus we obtain the following input demands for foreign heterogeneous intermediate goods

$$y_{F,t}(i) = \left( \frac{p_{F,t}(i)}{P_{F,t}} \right)^{\frac{-(1+\mu_F)}{\mu_F}} Y_{F,t}, \quad (26)$$

and the following price index

$$P_{F,t} = \left[ \int_0^1 p_{F,t}(i)^{\frac{-1}{\mu_F}} di \right]^{-\mu_F}. \quad (27)$$

### 2.2.3 Heterogeneous Intermediate Goods Producers

The technology for producing each heterogeneous intermediate good  $i \in [0, 1]$  is the standard constant returns to scale production function

$$y_{H,t}(i) + \frac{1-\omega}{\omega} y_{H,t}^*(i) = y_t(i) = A k_t(i)^\alpha (z_t l_t(i))^{1-\alpha}, \quad (28)$$

where  $k_t(i)$  and  $l_t(i)$  are the inputs of capital and labour, respectively, while  $y_H(i)$  and  $y_H^*(i)$  are the amounts of the home intermediate heterogeneous good  $i$  sold in the home country and in the foreign country<sup>7</sup>, respectively. Moreover  $z_t$  denotes the stationary technology shock that follows an AR(1) process

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}, \quad (29)$$

where  $E_t[z_t] = 1$  and  $\hat{z}_t = z_t - 1$  (in the deterministic steady state  $\bar{z} = 1$ ).

Next we find the cost function to simplify notationally the profit maximization problem. The cost minimization problem of the producer  $i$  in period  $t$  is as follows

$$c(y_t(i)) = \min_{k_t(i), l_t(i)} [r_t k_t(i) + w_t l_t(i)] \quad (30)$$

subject to (28), where  $r_t$  and  $w_t$  are the gross nominal rental rate of capital and the real wage, respectively. Solving (30) we get the following cost function

$$c(y_t(i)) = mc_t y_t(i), \quad (31)$$

where  $mc_t$  denotes marginal cost,  $mc_t(y_t(i)) = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{1}{z_t^{1-\alpha}} r_t^\alpha w_t^{1-\alpha}$ . Moreover, from the cost minimization problem we get the following condition for the optimal inputs employment ratio

<sup>7</sup>Note that  $y_{H,t}^*(i)$  is expressed in per capita units of foreign country thus it has to be multiplied by  $\frac{1-\omega}{\omega}$ .

$$\frac{r_t}{w_t} = \frac{\alpha}{1 - \alpha} \frac{l_t(i)}{k_t(i)}. \quad (32)$$

The producer of the home heterogeneous intermediate good  $i$  sells her products both to home homogeneous intermediate good producers and foreign homogeneous intermediate good producers. Since no two goods are the same, its producers operate in a monopolistically competitive market and have market power. We assume that the producers set their prices according to the Calvo scheme which introduces price stickiness into the model. Furthermore, the prices are sticky in a buyers currency, which is consistent with the incomplete short term pass-through. Since the marginal cost function is constant in  $y_t(i)$ , we can write down the separate profit maximization problems for goods sold in the home country and the foreign country. In each period, the producer of good  $i$ , while selling her goods in the home country  $y_H(i)$ , with probability  $(1 - \theta_H)$  receives a signal to adjust her price, otherwise the price evolves according to the following formula  $p_{H,t+1}(i) = p_{H,t}(i) \bar{\pi}$ , where  $\bar{\pi}$  denotes the deterministic steady state inflation in the home country. If the producer receives the signal for price reoptimization it sets the new price  $p_{H,t}^{new}(i)$  that maximizes her profits given by the following function

$$E_t \sum_{j=0}^{\infty} (\beta \theta_H)^j \Lambda_{t,t+j} \left( \frac{(1 + \tau_H) p_{H,t}^{new}(i) \bar{\pi}^j}{P_{t+j}} - mc_{t+j} \right) y_{H,t+j}(i) \quad (33)$$

subject to the demand function (24), where  $\Lambda_{t,t+j}$  denotes the intertemporal discount factor (consistent with the home country households problem), and  $\tau_H$  denotes the production subsidy<sup>8</sup>.

Similarly, the producer of good  $i$ , while selling her product in the foreign country, in each period with probability  $(1 - \theta_H^*)$  reoptimizes her price and with probability  $\theta_H^*$  the price evolves according to the following formula  $p_{H,t+1}^*(i) = p_{H,t}^*(i) \bar{\pi}^*$ , where  $\bar{\pi}^*$  denotes the deterministic steady state inflation in the foreign country. While reoptimizing the producer chooses  $p_{H,t}^{new,*}(i)$  that maximizes the following profit function

$$E_t \sum_{j=0}^{\infty} (\beta \theta_H^*)^j \Lambda_{t,t+j} \left( \frac{e_{t+j} (1 + \tau_H^*) p_{H,t}^{new,*}(i) (\bar{\pi}^*)^j}{P_{t+j}} - \frac{1 - \omega}{\omega} mc_{t+j} \right) y_{H,t+j}^*(i) \quad (34)$$

subject to the demand function (26), where  $e_{t+j}$  denotes the nominal exchange rate (price of the foreign currency in the home currency), and  $\tau_H^*$  denotes the foreign production subsidy.

<sup>8</sup>We assume that the subsidy is set by respective governments to eliminate the monopolistic distortion associated with positive markups, thus for  $d \in \{H, F\}$  we have  $\tau_d = \theta_d$  and  $\tau_d^* = \theta_d^*$ .



Define the real exchange rate as  $q_t = \frac{e_t P_t^*}{P_t}$ . Solving the problems (33) and (34) we get the following first order conditions

$$E_t \sum_j (\beta \theta_H)^j \Lambda_{t,t+j} \left( \frac{p_{H,t}^{new}(i) \bar{\pi}^j}{P_{t+j}} - \frac{1 + \mu_H}{1 + \tau_H} mc_{t+j} \right) y_{H,t+j}(i) = 0, \quad (35)$$

and

$$E_t \sum_{j=0}^{\infty} (\beta \theta_H^*)^j \Lambda_{t,t+j} \left( \frac{q_{t+j} p_{H,t}^{new,*}(i) (\bar{\pi}^*)^j}{P_{t+j}^*} - \frac{1 + \mu_H^*}{1 + \tau_H^*} \frac{1 - \omega}{\omega} mc_{t+j} \right) y_{H,t+j}^*(i) = 0. \quad (36)$$

Thus the producer sets its price so that discounted real marginal revenue is equal to discounted real marginal cost, in expected value. Note that if  $\theta = 0$  we obtain the standard condition that price equals marginal cost. The log-linearised equations can be found in Appendix A.

### 2.3 Government

A government uses lump sum taxes to finance government expenditure and production subsidies. The government's budget constraint in this economy is given by

$$G_t + \int_0^1 \tau_H p_{H,t}(i) y_{H,t}(i) di + \int_0^1 \tau_F p_{F,t}(i) y_{F,t}(i) di = T_t. \quad (37)$$

Since in our framework Ricardian equivalence holds there is no need to introduce government debt. Moreover, we assume that government expenditures are driven by a simple autoregressive process

$$G_{t+1} = (1 - \rho_g) \mu_g + \rho_g G_t + \varepsilon_{g,t+1}. \quad (38)$$

### 2.4 Central Bank

As it is common in the New-Keynesian literature, we assume that monetary policy is conducted according to a Taylor rule (we apply the concept of an extended Taylor rule, similar to the one assumed by Smets and Wouters (2003)) that targets deviations from the deterministic steady state inflation, GDP, the growth rate of inflation and the growth rate of GDP, allowing additionally for the interest rate smoothing

$$R_t = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\gamma_R} \left( \left( \frac{\pi_t}{\bar{\pi}} \right)^{\gamma_\pi} \left( \frac{\tilde{Y}_t}{\bar{Y}} \right)^{\gamma_y} \left( \frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} \right)^{\gamma_{dy}} \left( \frac{\pi_t}{\pi_{t-1}} \right)^{\gamma_{d\pi}} \right)^{1-\gamma_R} e^{\varphi_t}, \quad (39)$$

where  $\pi_t = \frac{P_t}{P_{t-1}}$ , and  $\bar{\pi}$ ,  $\bar{Y}$  denote the deterministic steady state values of inflation and GDP, respectively. It's worth noting that the Taylor rule plays a key role in bringing stability to the model and determining the reaction of the model economy to exogenous shocks<sup>9</sup>.

## 2.5 Market Clearing

In equilibrium, the goods markets, the assets markets and the production factors markets must clear.

The market clearing condition in the final goods market takes the form

$$c_t + x_t + G_t + \Psi(u_t)k_t = Y_t. \quad (40)$$

Note that we have included the market clearing condition in the intermediate goods markets using notation.

The market clearing condition in the production factors markets are

$$\int_0^1 l_t(i) di = l_t, \quad (41)$$

$$\int_0^1 k_t(i) di = k_t. \quad (42)$$

Since domestic bonds cannot be traded internationally and the governments budgets are balanced, the market clearing conditions for domestic bonds markets in both countries are

$$B_{t+1} = B_{t+1}^* = 0 \quad (43)$$

and the value of the total debt of both countries has to be zero

$$D_{H,t+1}^* + D_{F,t+1}^* = 0. \quad (44)$$

## 2.6 Balance of Payments and GDP

Using the market clearing conditions and the budget constraint we get the balance of payments equation

$$\int_0^1 \frac{1-\omega}{\omega} e_t (1 + \tau_H^*) p_{H,t}^*(i) y_{H,t}^*(i) di + \frac{e_t D_{H,t+1}^*}{R_t^* \kappa_t} = \int_0^1 (1 + \tau_F) p_{F,t}(i) y_{F,t}(i) di + e_t D_{H,t}^*. \quad (45)$$

Furthermore, to close the model, since there is GDP in the Taylor rule we need the formula for GDP, which has the following form

<sup>9</sup>For discussion see Carlstrom and Fuerst (2005).

$$P_t \tilde{Y}_t = P_t Y_t + \int_0^1 \frac{1-\omega}{\omega} e_t (1 + \tau_H^*) p_{H,t}^*(i) y_{H,t}^*(i) di - \int_0^1 (1 + \tau_F) p_{F,t}(i) y_{F,t}(i) di. \quad (46)$$

The log-linearised equations can be found in Appendix A.

### 3 Calibration and estimation

In order to evaluate the properties of the model against the data describing Polish and the Eurozone economies, we applied a mixture of calibration and estimation procedures. First, we calibrated a subset of parameters that can be easily extracted from the raw data or those resulting from the steady state considerations. Afterwards, we performed a Bayesian estimation of the other parameters, that mainly govern the business cycle volatility of the model.

#### 3.1 Calibration Procedure

The calibration of the parameters was based mainly on the data from the quarterly National Accounts, issued either by the Eurostat (in case of the Eurozone<sup>10</sup>), or by the Polish Central Statistical Office (GUS). As a measure of exports and imports we used the data from the Polish National Accounts, then we adjusted the Eurozone data, treating the resulting additional net trade with the rest of the world as government consumption<sup>11</sup>. Due to the lack of data on average hours worked, we used employment as a proxy for a measure of labor in the model. We used the data on total employment (domestic concept) from the Eurostat, in case of the Eurozone, and the data on employment from the Labor Force Study, in case of Poland. As a measure of wages, we used quarterly data on average wages in the national economy, in case of Poland (due to the lack of data on the compensation of employees for the whole period) and data on compensation of employees per person employed, in case of the Eurozone.

Table 1: The most important calibrated parameters of the model

Parameter	$\beta$	$\beta^*$	$\delta$	$\delta^*$	$\frac{1+\mu}{\mu}$	$\frac{1+\mu^*}{\mu^*}$	$\eta$	$\eta^*$	$\alpha$	$\alpha^*$
Value	0.99	0.99	0.017	0.017	2	2	0.61	0.99	0.33	0.33

The most important calibrated parameters of the model can be found in Table 1. The discount factors  $\beta$  and  $\beta^*$  were set at the same levels<sup>12</sup> of 0.99, which implies the annual long-term real interest rate of 4%, consistent with the average real interest rate (3-months interest rate deflated by the expected inflation, under assumption of a perfect foresight) in the Eurozone for the period 1995-2007. The physical capital depreciation rate  $\delta$  was set at

<sup>10</sup>The Eurozone is defined as EA-15 and includes: Belgium, Cyprus, Denmark, Ireland, Greece, Spain, France, Italy, Luxembourg, Malta, Netherlands, Austria, Portugal, Slovenia, and Finland.

<sup>11</sup>In a two country framework one need to decide how to deal with the trade with rest of the world. Treating it as government consumption in our framework is relatively nondistortionary, as it only affect the steady state government consumption share and is roughly in line with the approach of Chari, Kehoe, and McGrattan (2007).

<sup>12</sup>In order to avoid the steady state effects of monetary policy regime change, we used the same discount factor for both economies which was set at the level consistent with the Eurozone.

7% annually in both countries. The elasticity of production with respect to capital,  $\alpha$  was set at 0.33 in both countries, in line with most of the DSGE literature. The long-term inflation rate was set at 2.5% annually for both economies. The share of the population of Poland in the total population of the Eurozone and Poland,  $\omega$  was set at 0.107, on the basis of the data from the Eurostat.

The parameters  $\mu$  and  $\mu^*$  were set at 1 in both countries, implying the Armington elasticity of substitution  $\frac{1+\mu}{\mu} = 2$ , consistently with the evidence given by Ruhl (2005) and discussion presented in McDaniel and Balistreri (2003). The home bias parameters,  $\eta$  and  $\eta^*$  were set at 0.614 and 0.9903 respectively, reflecting the export to absorption ratios for the period 2004 – 2007 in case of Poland and 1995 – 2007 in case of the Eurozone. The steady state consumption shares (in absorption) were set at 0.609 and 0.573 in Poland and the Eurozone, respectively. The corresponding figures for investment shares are 0.206 and 0.2095. We treat the government expenditures share as a residual. The absorption to GDP ratios were set at 1.03 and 0.99 in Poland and the Eurozone. We set the debt share at 0.46, in line with the average ratio of total external debt of the Polish economy to GDP for the years 2005 – 2007. Some other parameters of the model, that were not estimated were calculated on the basis of the steady-state relations of the model.

### 3.2 Estimation Procedure

All data used in the estimation of the model were expressed in quarterly frequency and adjusted for seasonality (except for the interest rates) using Demetra package and expressed in constant prices from the year 2000. As the model does not distinguish between different price indicators and monetary policy is aimed at stabilizing the consumer price inflation, we decided to express all real variables in terms of consumption prices either in Poland or in the Eurozone. We also normalized the variables by the population shares  $\omega$  and  $1 - \omega$  in each quarter<sup>13</sup>. Then, the logarithms of all variables were detrended using the Hodrick-Prescott filter (see Hodrick and Prescott, 1997), with the standard parameter for the quarterly data  $\lambda = 1600$ , and expressed as a log-difference of a given variable and HP-trend, which is consistent with the log-linearisation of the model.

Our approach was to keep the basic structure of the model relatively simple and to use a relatively small number of fundamental stochastic shocks to describe the cyclical fluctuations of the economy (we used technology, government consumption, monetary policy, and labor supply<sup>14</sup> shocks for

<sup>13</sup>As there are no consistently measured quarterly data for the Eurozone, we extrapolated the annual data using constant quarterly growth rates within a year, assuring that the data for the beginning of the year are the same as measured by annual data.

<sup>14</sup>Especially in case of Poland, it is very hard to model the labor market variables using a relatively simple concept of labor market equilibrium, as in the presented model. So

both countries and risk premium shocks). But this approach would limit the information that is used in the estimation, as the number of observed variables needs to be equal to the number of stochastic shocks. So, in the estimation of the parameters, we included additional information from other variables, assuming that they are observed with an iid noise. As primary variables (observed without noise), we used the following time series for the period *IIIQ1996 – IVQ2007* for both Poland and the Eurozone: GDP, government expenditure, consumer price inflation, employment and the interest rate differential (in gross terms, compatible with the definition of  $\hat{R} - \hat{R}^*$  in the model). The set of additional variables (observed with noise) includes (for both economies): consumption, investments and trade indicators (both exports and imports, expressed in Polish currency).

As the model is log-linearised, it can be expressed as a state-space representation and its likelihood can be evaluated against data via the Kalman filter. We decided to use the Bayesian approach to estimate the model parameters, since it allows to use additional information, that can be provided via the prior distribution of the parameters. The expression for the likelihood function cannot be found analytically, however the posterior distribution of the model parameters can be estimated via Monte Carlo Markov Chain (MCMC) algorithm, proposed by Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) and Hastings (1970). We used the Dynare package in order to solve, estimate and simulate the model (see e.g. Juillard (1996)).

After finding the mode of the posterior distribution (using the Chris Sims *csminwel* procedure that applies a quasi-Newton method with the BFGS update of the estimated inverse Hessian, robust against cliffs, i.e. hyperplane discontinuities) we applied the Metropolis-Hastings MCMC algorithm with 50000 replications (to assure that the MCMC algorithm converges) in two blocks in order to compute the posterior distribution of the model parameter. The acceptance rates were ca. 24%, within the band 23%-25% recommended in the literature. On the basis of the univariate and multivariate convergence diagnostic of Brooks and Gelman (1998) indicating that the MCMC algorithm converged, we dropped 10% of draws for the purpose of construction of the posterior probability distributions.

When choosing the parameters of the prior distributions, we used the results of Smets and Wouters (2003), Adolfson, Laséen, Lindé, and Villani (2005) and Kolasa (2008) and, to some extent, some pre-estimation exercises. To simplify notation we denoted  $\frac{\Psi'(1)}{\Psi''(1)}$  as  $\Psi$ . The chosen parameters of the prior distribution are presented in Table 2. In most cases we have not distinguished between Poland and the Eurozone, except for  $\iota$ 's, Calvo probabilities<sup>15</sup>  $\theta$ 's and parameters of the Taylor rule. As the installation

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we decided to add labor supply shocks, in order to allow the model to have a chance to describe the labor market behaviour in line with the data.

<sup>15</sup>Note, that here by Calvo probability we mean probability that a price is not adjusted in a given period. Thus the higher Calvo probability the less often prices are adjusted.

Table 2: Basic description of the prior and posterior distributions

Parameter	Prior distribution			Posterior distribution		
	type	Mean	St. Dev.	Mode	Mean	St. Dev.
$\iota$	norm	0.10	0.025	0.087	0.088	0.023
$\iota^*$	norm	0.30	0.025	0.347	0.346	0.024
$\Psi$	norm	0.20	0.075	0.240	0.242	0.071
$\Psi^*$	norm	0.20	0.075	0.301	0.305	0.068
$\gamma$	norm	4.00	0.250	4.056	4.053	0.246
$\gamma^*$	norm	4.00	0.250	4.053	4.053	0.245
$\theta_H$	beta	0.60	0.080	0.443	0.446	0.053
$\theta_F^*$	beta	0.70	0.080	0.500	0.502	0.043
$\theta_F$	beta	0.60	0.080	0.791	0.762	0.082
$\theta_H^*$	beta	0.70	0.080	0.722	0.708	0.068
$\chi$	norm	0.05	0.030	0.078	0.078	0.026
$\sigma$	norm	2.00	0.300	3.144	3.157	0.242
$\sigma^*$	norm	2.00	0.300	1.416	1.467	0.214
$\gamma_R$	beta	0.82	0.020	0.714	0.714	0.019
$\gamma_\pi$	norm	1.25	0.070	1.289	1.305	0.084
$\gamma_y$	norm	0.40	0.050	0.464	0.459	0.051
$\gamma_{d\pi}$	norm	0.20	0.030	0.200	0.199	0.030
$\gamma_{dy}$	norm	0.10	0.020	0.108	0.107	0.020
$\gamma_R^*$	beta	0.80	0.020	0.760	0.758	0.020
$\gamma_\pi^*$	norm	1.30	0.080	1.497	1.497	0.071
$\gamma_y^*$	norm	0.30	0.060	0.187	0.210	0.061
$\gamma_{d\pi}^*$	norm	0.20	0.030	0.205	0.202	0.030
$\gamma_{dy}^*$	norm	0.10	0.020	0.099	0.098	0.020
$\rho_z$	beta	0.95	0.010	0.943	0.942	0.011
$\rho_z^*$	beta	0.95	0.010	0.943	0.941	0.010
$\rho_G$	beta	0.85	0.100	0.632	0.642	0.115
$\rho_G^*$	beta	0.85	0.100	0.934	0.918	0.026
$\rho_l$	beta	0.60	0.100	0.765	0.753	0.056
$\rho_l^*$	beta	0.60	0.100	0.743	0.731	0.053
$\rho_{a\kappa}$	beta	0.60	0.100	0.556	0.543	0.062
$\sigma_z$	inv gamma	0.06	$\infty$	0.012	0.012	0.002
$\sigma_z^*$	inv gamma	0.005	$\infty$	0.004	0.004	0.0004
$\sigma_\varphi$	inv gamma	0.02	$\infty$	0.009	0.010	0.001
$\sigma_\varphi^*$	inv gamma	0.006	$\infty$	0.002	0.002	0.0004
$\sigma_G$	inv gamma	0.02	$\infty$	0.016	0.017	0.002
$\sigma_G^*$	inv gamma	0.02	$\infty$	0.018	0.018	0.002
$\sigma_l$	inv gamma	0.04	$\infty$	0.040	0.041	0.005
$\sigma_l^*$	inv gamma	0.01	$\infty$	0.012	0.012	0.002
$\sigma_{a\kappa}$	inv gamma	0.05	$\infty$	0.023	0.025	0.004

of the new investments goods in Poland could be more costly, due to more constricted regulations we chose lower value for  $\iota$  than for  $\iota^*$ . Also, due to less stringent product market regulations in Poland, we pick slightly lower Calvo probabilities for Poland than for the Eurozone. Additionally, taking into account the results of Kolasa (2008), we picked such prior distributions of the parameters of the Taylor rule, that the policy rule in the Eurozone is slightly more inflation oriented than in Poland. The standard deviations of the stochastic shocks were set on the basis of the pre-estimation exercise.

For standard deviations of the noise components of the additional observed variables, we assumed that the variability of the noise component equals 1% of the overall variability of a given variable.

### 3.3 Parameters

The results of the estimation procedure are shown in Table 2. Additionally, we plotted the prior and posterior distributions of all estimated parameters - see Figure 1, including the standard deviations of the noisy components of the additional variables<sup>16</sup>.

The results of the estimation indicate that our assumption of the relative size of the parameter governing the cost of capital adjustment was correct. Also, the additional information from the data have not changed strongly the prior distribution of the labor supply elasticity. The estimated coefficients of the Calvo parameters indicate that the degree of nominal rigidity in Poland is lower than in the Eurozone, especially in case of goods sold domestically. The estimated Calvo probabilities for the Eurozone are roughly in line with the results of Adolfson, Laséen, Lindé, and Villani (2005).

The information in the data series significantly changed the estimate of the inverse of the elasticity of intertemporal substitution in Poland - the estimated parameter  $\sigma = 3.2$  is much higher than the mean of the prior, which was set at 2.

The estimated interest rate smoothing coefficient in the Taylor rule for Polish economy proved to be lower than expected,  $\gamma_R = 0.71$ , although within the range usually obtained in the literature. Additionally, the responsiveness of the interest rate to the consumer price inflation and GDP is slightly higher than assumed, both in case of Poland and the Eurozone. The estimated degree of the interest rate smoothing of the ECB  $\gamma_{R^*} = 0.76$ , is slightly lower than the usual estimates (see e.g. Smets and Wouters, 2003 or Adolfson, Laséen, Lindé, and Villani, 2005). The estimated Taylor rules are as follows

$$\hat{R}_t = 0.71\hat{R}_{t-1} + 0.29[1.31\hat{\pi}_t + 0.46\hat{Y}_t + 0.11d\hat{Y}_t + 0.2d\hat{\pi}_t] + \varphi_t, \quad (47)$$

in case of the NBP, and

$$\hat{R}_t^* = 0.76\hat{R}_{t-1}^* + 0.24[1.5\hat{\pi}_t^* + 0.21\hat{Y}_t^* + 0.1d\hat{Y}_t^* + 0.2d\hat{\pi}_t^*] + \varphi_t^*, \quad (48)$$

in case of the ECB.

The estimated persistence of the technology shocks is very similar in Poland and in the Eurozone, roughly equal to 0.94. The estimation revealed that the persistence of the government spending shocks is much higher in the Eurozone than in Poland, in line with the economic intuition. Also, the persistence of the labor supply shocks proved to be higher in case of Poland,

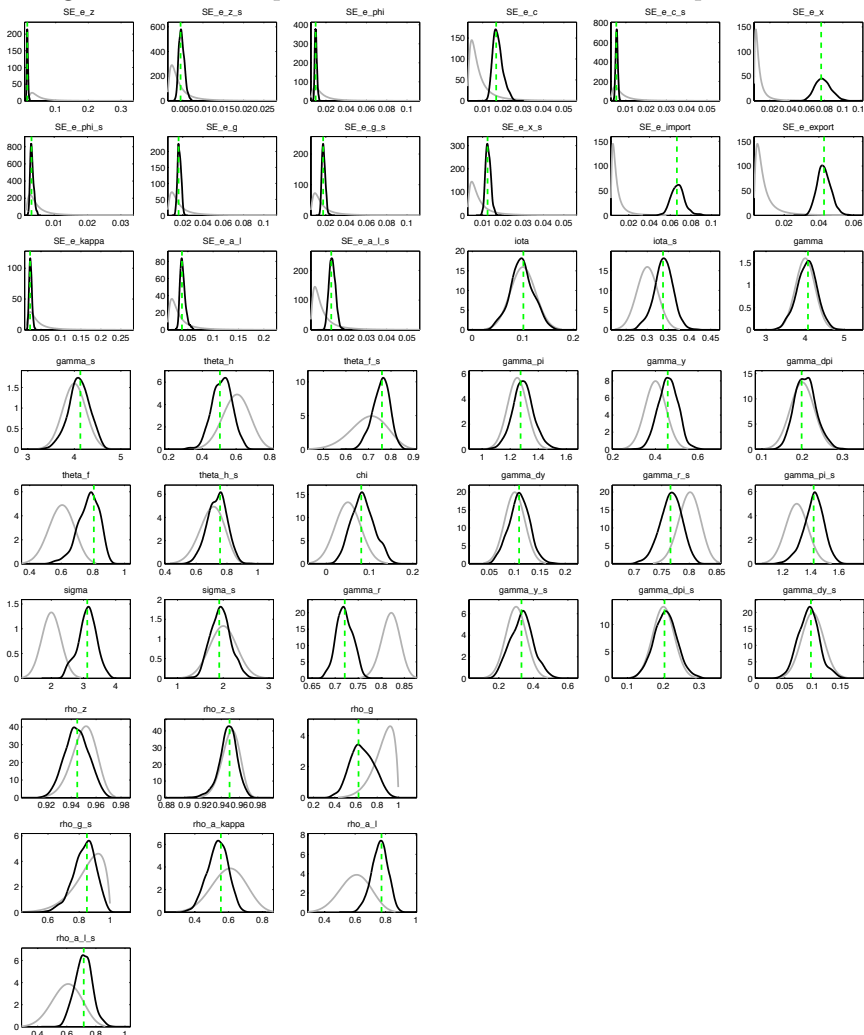
<sup>16</sup>These are the plots of: SE.e.c, SE.e.c.s, SE.e.x, SE.e.x.s, SE.e.import, SE.e.export.



reflecting the features of the transforming economy. The persistence of the risk premium shocks proved to be relatively small, also in line with the economic intuition<sup>17</sup>.

The estimated volatilities of the shocks governing the evolution of the economy (reported in last rows of Table 2) reveal that the volatility of the technology shocks is much larger in Poland, again in line with the economic intuition and with the evidence from the relative volatility of output of the Polish economy compared to the Eurozone. The same applies for the monetary policy shocks and the labor supply shocks. On the other hand, the volatility of the government spending shocks is slightly higher in the Eurozone, than in Poland.

Figure 1: Prior and posterior distributions of the model parameters



<sup>17</sup>Taking into account the fact that adjustments of the exchange rate are relatively fast, it is not surprising that agents respond relatively quickly to the exogenous changes in the interest rate disparities.

### 3.4 Model's Data Fit

In order to evaluate the ability of the model to replicate the features of the data from the Polish and the Eurozone economies, we compared the moments of the model generated variables against the moments of the data. Instead, we could have had used the theoretical moments of the model's variables, but these describe the large sample properties of the model, whereas to calculate the moments of the data we can use only ca. 50 observations. In order to overcome this issue we decided to simulate the model behaviour in a short sample. So, we simulated the model using random draws of the stochastic shocks<sup>18</sup> for 152 periods and then dropped the first 100 observations<sup>19</sup>, calculating the moments for only 52 observations. We replicated this procedure 10000 times (for different, independent draws of the stochastic shocks) and computed the averages of the model generated moments in order to assure that the calculated moments are history-independent.

Table 3 shows the results of this procedure. The upper panel shows

Table 3: Moments of the model generated variables against the data

	Volatility		Correlation with GDP		Persistence	
	data	model	data	model	data	model
Poland						
$\tilde{Y}$	0.020	0.025	1.000	1.000	0.403	0.597
$c$	0.010	0.011	0.611	0.245	0.806	0.738
$x$	0.089	0.046	0.874	0.481	0.800	0.988
$\pi$	0.010	0.022	-0.276	-0.187	-0.279	0.696
$w$	0.012	0.026	0.464	0.517	0.662	0.536
$R$	0.021	0.019	0.282	-0.445	0.849	0.864
$l$	0.015	0.012	0.601	0.250	0.928	0.750
$q$	0.061	0.043	0.118	0.706	0.813	0.382
<i>export</i>	0.048	0.048	0.337	0.792	0.812	0.432
<i>import</i>	0.062	0.018	0.517	0.259	0.765	0.654
Eurozone						
$\tilde{Y}^*$	0.006	0.010	1.000	1.000	0.817	0.843
$c^*$	0.006	0.008	0.842	0.329	0.868	0.825
$x^*$	0.027	0.035	0.758	0.513	0.935	0.981
$\pi^*$	0.003	0.007	0.288	-0.138	0.215	0.396
$w^*$	0.004	0.013	0.617	0.766	0.646	0.510
$R^*$	0.006	0.004	0.635	-0.660	0.896	0.838
$l^*$	0.005	0.004	0.805	0.459	0.948	0.664

the results for the Polish economy, and the lower panel for the Eurozone economy. The first two columns show the volatilities of the model generated variables against the data (measured by the standard deviations), the middle

<sup>18</sup>As was mentioned earlier, the additional shocks of the model - noise in additional observables - were used only for the purpose of the estimation, so in the simulations performed on the model, we turned off these shocks.

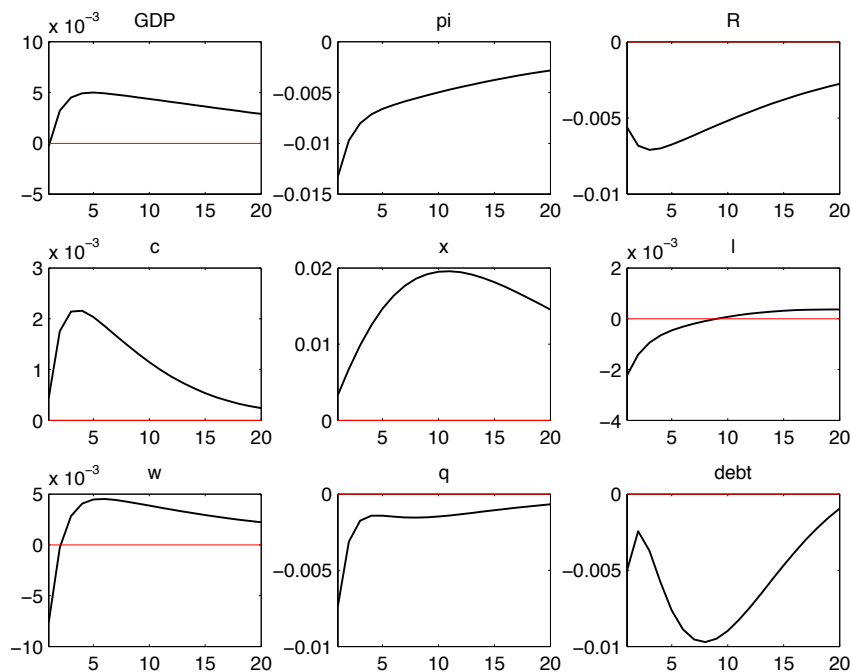
<sup>19</sup>As the simulation starts from the deterministic steady state, we dropped some observations to assure that we calculate the moments of variables not being biased by being too close to the steady state.

two columns show the cyclicity of variables, as measured by correlations with  $GDP$  ( $GDP^*$  in case of the Eurozone). The last columns show the persistence of the model generated variables and the data - measured by autocorrelation.

The model generates too much volatility of  $GDP$ , both in Poland and in the Eurozone. The same is true for inflation and real wage (and investment in case of the Eurozone economy). The model correctly reproduces, for both economies, the volatilities of consumption, the nominal interest rate, and exports. The volatilities of investment in Poland and employment in both economies, as well as imports were underestimated by the model.

Our model generates too little comovement with output in case of consumption, investments, employment and imports in case of Poland. The same is true for consumption and employment in case of the Eurozone. The cyclicity of the real exchange rate is largely exaggerated by the model. The model predicts countercyclical inflation in the Eurozone, whereas it is slightly procyclical in the data. Also the cyclicity of the nominal interest rate is different in the model and data for both economies - in the data the nominal interest rate is procyclical (slightly procyclical in case of Poland), whereas the model predicts the nominal interest rate is countercyclical (the procyclicality of interest rates in the data is rather not intuitive and could be an artifact of the short sample used in the analysis). Generally the model

Figure 2: Impulse response to the asymmetric technology shock



reproduces well the persistence of the analysed variables - excluding the real exchange rate, exports, and especially inflation.

Summing up, the model fits the data relatively well, although not perfectly. The main problems are: exchange rate (even with the equity premium puzzle shocks the model cannot generate the properties of this variable, but this is rather common to this kind of methodology - see e.g. Chari, Kehoe, and McGrattan, 2002), cyclicity of the interest rates, and cyclicity of inflation (the latter in case of the Eurozone). Overall we think that the performance of the model is rather good.

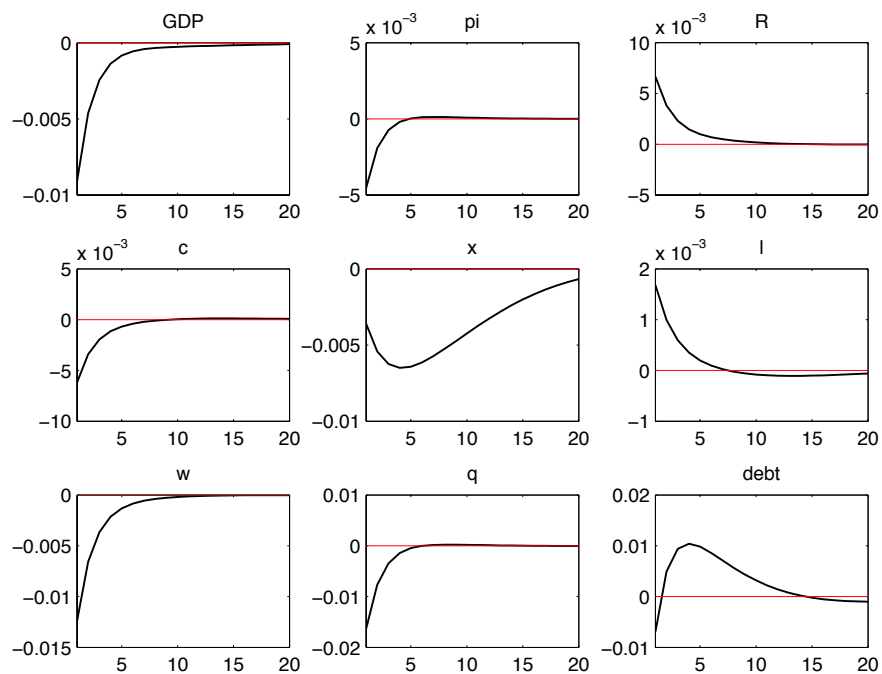
### 3.5 Impulse Response Functions

In order to understand the dynamic properties of the model, we calculated the impulse response of the model to the most important shocks of the model - asymmetric technology shock and asymmetric monetary policy shock (that occurred in the Polish economy).

#### 3.5.1 Asymmetric Technology Shock

After an unanticipated asymmetric technology shock (hitting the Polish economy, see Figure 2) the model predicts the prolonged increase of both consumption and investments, the latter generating the increase of capital stock. The decrease of the domestic prices translates into falling inflation

Figure 3: Impulse response to the asymmetric monetary policy shock



and a decline of the nominal interest rates. Real wages decrease on impact, but then quickly rise above the steady state and stay there for a while. The technology shock proved to be labor-saving, resulting in a drop of employ-

ment for the first 10 quarters. This effect is a standard feature of the sticky price models, which is highly debated in the literature<sup>20</sup>. After a couple of quarters employment recovers and at some point it even goes over its steady state value. With lower employment (and higher labor productivity) and higher capital, GDP rises and then slowly returns to its steady state.

Higher level of activity in the Polish economy translates into a decrease of foreign debt - domestic agents use the period of higher activity of the economy and higher income to pay back some of the debt they have against the foreign agents. Higher productivity in the domestic economy, results in a decline of the real exchange rate (i.e. exchange rate appreciates in real terms). After about 6 – 8 years after the shock the economy converges back to the steady state.

### 3.5.2 Asymmetric Monetary Policy Shock

Figure 3 presents the response of the model economy to an unanticipated asymmetric domestic monetary policy shock. Initially, the domestic interest rate increases, then the Taylor rule kicks in and the interest rate slowly reverts to the steady state.

In response to a higher nominal interest rate, we observe a decline in inflation, consumption and investment. Although real wage also decreases, we observe an increase of employment, since income effect dominates the substitution effect. In spite of the latter, GDP falls, due to the decline of capital. After the initial decline of debt, households start to borrow from abroad to smooth out consumption. In reaction to an increase in the interest rates differential, the exchange rate appreciates (i.e. drops) in real terms.

When we compare the reaction of the model to the technology shock (see Figure 2) and the monetary policy shock (see Figure 3), we can observe that the economy stabilizes much faster in the latter case. This feature is relatively intuitive - the technology shock is an example of a supply shock, generating more persistent response of the economy. On the other hand - monetary policy shock, as a pure demand shock, generates much less persistence.

<sup>20</sup>For discussion see for example Christiano, Eichenbaum, and Vigfusson (2003).

## 4 Results

In this section we analyse the business cycle behaviour of the most important macroeconomic variables of the Polish economy in the presence of shocks in two regimes:

- *Autonomous Monetary Policy* (denoted as OUT), monetary policy in Poland is conducted by the National Bank of Poland and the Taylor rule describing this policy is given by equation (47), while monetary policy in the Eurozone is conducted by the ECB and the Taylor rule that describes this policy is given by equation (48). The nominal exchange rate is not fixed and adjusts freely to the market conditions.
- *Common Monetary Policy* (denoted as IN), monetary policy both in Poland and the Eurozone is conducted by the ECB according to the Taylor rule that is the same as the ECB Taylor rule estimated from the data, except for it assigns the weight  $\omega_T = \frac{\hat{Y}^{PL}}{\hat{Y}^{EUR} + \hat{Y}^{PL}} = 2,97\%$  to the Polish variables:

$$\begin{aligned} \hat{R}_t^* = & 0.76\hat{R}_{t-1}^* + (1 - 0.76)[1.5(\omega_T\hat{\pi}_t^{ass} + (1 - \omega_T)\hat{\pi}_t^*) + \\ & + 0.21(\omega_T\hat{Y}_t + (1 - \omega_T)\hat{Y}_t^*) + 0.1(\omega_T\hat{dY}_t + (1 - \omega_T)\hat{dY}_t^*) + \\ & + 0.2(\omega_T\hat{d\pi}_t + (1 - \omega_T)\hat{d\pi}_t^*)] + \varphi_t^*. \end{aligned} \quad (49)$$

The nominal exchange rate is fixed and cannot be changed  $e_t = \bar{e}$ . We want to stress that in this simulation we do not eliminate the risk premium volatility as a source of the interest rate differential between domestic and foreign households. In our model the risk is associated with fluctuations of the domestic debt rather than fluctuations of the exchange rate. Thus the accession to the Eurozone does not eliminate the effect that this risk has on the interest rate differential.

Given the differences between the two regimes there are four important factors that may affect the Polish economy after joining the Eurozone: (1) the Taylor rule that is more inflation oriented; (2) less variability hitting the Polish economy, since the Polish monetary policy shock is replaced with the Eurozone monetary policy shock (which standard deviation is 3 times smaller); (3) the monetary policy rule focused on the whole Eurozone economy rather than the Polish economy; and (4) fixed nominal exchange rate.

To analyse the differences between the two regimes we run 50,000 simulations for 1000 periods each. Using the results from the simulations we compare the business cycle behaviour of the most important macroeconomic variables and we compute the change of the consumer's welfare in Poland after the Eurozone accession.

#### 4.1 GDP and Inflation Variability

In our simulations we calculated standard deviations of the main macroeconomic variables. The results are presented in Table 4. Most variables are more volatile under the Common Monetary Policy, which is mostly due to the fact it fits the Polish economy less than the Autonomous Monetary Policy. The increase in volatility would be even bigger if it was not for this factor out-weighs the impact of the replacement of the domestic monetary policy shock with the Eurozone monetary policy shock (which is five times less volatile). The only exception is inflation, which is less volatile under the Common Monetary Policy than the Autonomous Monetary Policy. In this case there are three factors that decrease the volatility of inflation. First, the Taylor rule under the Common Monetary Policy is more inflation oriented than the Taylor rule under the Autonomous Monetary Policy. Second, the domestic monetary policy shock under the Autonomous Monetary Policy regime is more volatile than the foreign monetary policy shock under the Common Monetary Policy, thus adds more volatility to the economy and it mostly affects inflation. Third, fixing the nominal exchange rate stabilizes inflation in the home country. This result is somewhat different than Karam, Laxton, Rose, and Tamirisa (2008), who find that the volatility of both GDP and inflation increases after a new member country joins the Eurozone, whereas we find that volatility of GDP increases but inflation becomes more stable.

Table 4: Variability of the Polish business cycle

Variables	<i>Autonomous MonetaryPolicy</i>	<i>Common MonetaryPolicy</i>	Change
GDP	0.0262	0.0272	+4%
Inflation	0.0242	0.0096	-60%
Interest Rate	0.0211	0.0294	+39%
Consumption	0.0133	0.0225	+69%
Labor	0.0130	0.0142	+9%

One might also wonder why the interest rate after the Euro adoption is more volatile. This is mostly due to the fixing of the nominal exchange rate. Note, that in the Common Monetary Policy regime, the differential of the interest rates (the UIP condition, see equation (A.13)) depends on the risk premium, the expected change in the real exchange rate and the difference in the expected inflation. But, in the Common Monetary Policy regime the UIP condition is replaced with the following condition (in the log-linearised version)

$$\hat{R}_t - \hat{R}_t^* = \hat{\kappa}_t. \quad (50)$$

Thus, the shocks to the risk premium affect only the interest rate differential, whereas with floating exchange rates some of the impact is cushioned by the exchange rate (which absorbs some of the volatility). Also greater volatility of the interest rate increases the volatility of consumption, which explains why the volatility of consumption increases by more than the volatility of GDP.

## 4.2 Welfare Cost

The results presented in the previous subsection do not provide quantitative measure of the cost of losing autonomous monetary policy associated with joining the Eurozone. Thus, we expressed the cost in terms of the consumer welfare. We ask how much consumption households would be willing to give up in order to stay indifferent between joining and not joining the Eurozone. This corresponds to calculating the compensating variation associated with the full elimination of the Autonomous Monetary Policy regime. Welfare analysis follows the method of Lucas (1987).

We use our simulations to compute the consumers welfare in both regimes of

$$W^{OUT} = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t^{OUT}, l_t^{OUT}, \zeta_t) \right], \quad (51)$$

$$W^{IN} = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t^{IN}, l_t^{IN}, \zeta_t) \right]. \quad (52)$$

Next we compute what percentage (denoted by  $\lambda$ ) of every period consumption, consumers would have to give up to be indifferent between the regimes. To find  $\lambda$  we solve

$$W^{OUT}(\lambda) = W^{IN}, \quad (53)$$

where  $W(\lambda) = E_0 [\sum_{t=0}^{\infty} \beta^t u((1 - \lambda)c_t, l_t, \zeta_t)]$ . The details on how  $\lambda$  is computed are presented in Appendix B. We find that  $\lambda = 0.068\%$ . This means that losing autonomous monetary policy associated with joining the Eurozone would have the same effect as a decrease in consumption by 0.068% in every period. Note, that it does not mean that the authors expect consumption in Poland to decrease permanently by 0.068% after the Eurozone accession.

This result is a little bit smaller than the 0.25% estimate of Lopes (2007), but her result might be biased since, as far as we understand, she ran only one simulation in each regime (one realization of random shocks). It seems that in order to deal with this problem she extended the simulation for many periods (1000), but we are not entirely convinced that extending the length of the simulation solves the problem of the bias. So, instead, we ran 50000 simulations, as we noticed that in our case only a large number of simulations guarantees convergence of the welfare result.



We can also express this welfare cost in period zero consumption rather than permanent consumption. In order to do this we compute what percentage (denoted as  $\lambda_0$ ) of period zero consumption households would have to give up to be indifferent between the regimes. To find  $\lambda_0$  we solve

$$W^{OUT}(\lambda_0) = W^{IN}, \quad (54)$$

where  $W(\lambda_0) = E_0 [u((1 - \lambda_0)c_0, l_0, \zeta_0) + \sum_{t=1}^{\infty} \beta^t u(c_t, l_t, \zeta_t)]$ . We find that  $\lambda_0 = 6.17\%$ . This means that losing autonomous monetary policy associated with joining the Eurozone would have the same effect as a 6.17% (one time) decrease in period zero consumption. Again, note that it does not mean that the authors expect consumption in Poland to fall by 6.17% in the period after the Eurozone accession.

To get a grasp on the results note, that welfare is computed using the utility function in (3), thus it depends on consumption and labor. Hence, we need to analyse behaviour of consumption and labor in both regimes. Note from Table 4, that the volatility of both consumption and labor is higher in the Common Monetary Policy regime than in the Autonomous Monetary Policy regime. So given our utility function it clearly must translate into lower welfare in the Common Monetary Policy regime than in the Autonomous Monetary Policy regime. We also computed the welfare cost of the business cycle (in terms of the deterministic steady state consumption). We found that the welfare cost of business cycle in the Autonomous Monetary Policy regime is equal to 0.029%, whereas in the Common Monetary Policy regime is equal to 0.097%. These results are just a little bit higher than the estimates of Lucas (1987) for the US economy, who calculates the cost of the business cycle at 0.01%, but the US economy is more stable. Furthermore, Storesletten, Telmer, and Yaron (2001) use OLG framework to point out that the welfare cost of business cycle might be sensitive to the persistence of the variables. We do not use the OLG framework, nevertheless, in our model the persistence of both consumption and labor is roughly in line with the data.

We want to stress that in our model wages are not sticky, which translates into lower welfare cost of business cycles. Including the sticky wages in our model could increase the welfare cost of the Eurozone accession. Unfortunately, our way of simulating the welfare effect of joining the Eurozone precludes adding the wage stickiness to the model. This would be an interesting extension of our work. Another extension worth considering might be the extension is going beyond the representative agent framework, which has a potential for generating higher costs of business cycles. The heterogeneity alone might not be enough though, since the results of Schulhofer-Wohl (2008) show that adding the risk aversion heterogeneity among consumers with complete markets (full insurance against risk) actually decreases the costs of business cycle fluctuations. On the other hand, after the accession to the Eurozone, the volatilities of shocks hitting the Polish economy may

fall and their correlations with the Eurozone counterparts may increase<sup>21</sup>. The inclusion of these possible changes into our simulations should result in lower estimates of costs of losing autonomous monetary policy.

### 4.3 Decomposition of Volatility Changes

In this subsection we decompose the change in volatility of the main macroeconomic variables into separate factors related to the Eurozone accession presented on page 30. We stress that the simulations run here are used only for the purpose of isolating the effects of different factors that are involved in joining the Eurozone. In order to achieve the desired isolation, while making those simulations, we make counter-factual assumptions.

First, as the starting point for the decomposition of the variance we use the results from the simulation Out. These results are presented in Table 6 in the column denoted as “Simulation 1”.

Second, we compute the effect of the change of the Taylor rule parameters. The Eurozone Taylor rule differs from the Polish Taylor rule in two important ways: (1) it is more inflation oriented; and (2) the extend of the of the interest rate smoothing is higher. To isolate this effect we run the following simulation. We keep everything exactly the same as in the first simulation, but the parameters of the NBP Taylor rule. We replace the parameters of the NBP Taylor rule with the parameters from the ECB Taylor rule. This experiment allows us to say how the standard deviations of the main macroeconomic variables would change if the NBP ran autonomous monetary policy (responding to the Polish variables only), but with the same parameters as the ECB. The results of this simulation are presented in Table 6 in the column denoted as “Simulation 2”. To see the effect of this change compare the results from the first and second simulations. This change leads to lower variability of inflation and greater variability of GDP. This is not surprising given that the Taylor rule changes from more GDP oriented to more inflation oriented. Greater variability of GDP translates into greater variability of the real side of the economy, i.e. the standard deviations of consumption and labor increase. Also, greater interest rate smoothing combined with more inflation oriented monetary policy and smaller variability of inflation reduces the standard deviation of the interest rate.

Table 5: The decomposition of the volatility changes

Variables	Simulation 1	Simulation 2	Simulation 3	Simulation 4
GDP	0.0262	0.0317	0.0286	0.0272
Inflation	0.0242	0.0152	0.0129	0.0096
Interest rate	0.0211	0.0126	0.0106	0.0294
Consumption	0.0133	0.0158	0.0138	0.0225
Labor	0.0130	0.0133	0.0132	0.0142

<sup>21</sup>In the study we assumed no correlation between shocks in the Eurozone and Poland.

Third, we analyse the effect of the replacement of the domestic monetary policy shock with the foreign monetary policy shock. Thus, we make only modification comparing to the second simulation. We replace the value of the variance of the monetary policy shock (estimated for the Polish economy) with the corresponding value from the Eurozone. The purpose of this exercise is to isolate the effect of smaller volatility of the monetary policy shock in the Eurozone than in Poland. The results of this simulation are presented in Table 6 in the column denoted as “Simulation 3”. This change reduces the volatility of the economy, which is pretty intuitive, since the standard deviation of the monetary policy shock declines from 0.01 to 0.002.

Finally, we isolate the effects of fixing of the exchange rate combined with the adoption of the common monetary policy rule - as shown in equation (49)<sup>22</sup>. Fixing of the exchange rate has two effects for the economy: on the one hand it means losing an instrument that can cushion some external shocks to the economy (for example the shock to foreign demand for domestic output) but on the other hand the exchange rate risk affects negatively both exporters and importers. Therefore, depending on the strength of these two effects fixing the exchange rate may reduce or increase the volatility of the economy. While the effect of the fixing of the exchange rate is uncertain the effect of the adoption of the common monetary policy rule is clear. Since the common monetary policy rule reacts to the extended Eurozone variables rather than the Polish ones, it is less fit to stabilize the Polish economy. Thus, the adoption of the common monetary policy rule increases the volatility of the Polish economy<sup>23</sup>. Summing up, using theory we cannot determine the direction of the changes created by these effects and we have to rely on numerical simulations. The results of this simulation are presented in Table 6 in the column denoted as “Simulation 4”. In fact this is the final change, thus this simulation is exactly the same as the simulation IN in the previous subsection. These results show that the volatility of GDP decreases and some of its components become more stable and some less stable. In particular the volatility of consumption, investment, and imports goes up by, respectively, 63%, 21%, and 113% and the volatility of exports goes down by 53% (as the volatility of the real exchange rate declines by 72%). Also, the volatility of inflation decreases. On the other hand, the results show that the exchange rate absorbs some of the volatility of the interest rate, thus fixing it leads to an increase of the volatility of the interest rate differential (due to risk premium), which in turn translates into higher volatility of the domestic interest rate. Moreover, since the volatility of consumption and labor is higher, the relative impact of higher volatility the interest rate on consumption and labor is stronger than the impact of smaller volatility of GDP.

<sup>22</sup>Due to technical reasons - violation of the Blanchard-Khan conditions - we were not able to separate these two effects.

<sup>23</sup>This claim is also supported by the result of the next section where we show how the change of weight of the Polish economy in the common monetary policy rule changes the volatility of Polish variables.

#### 4.4 Sensitivity Analysis

In this subsection we analyse the effect of an increase of the weight of Polish variables in the extended Eurozone Taylor rule from the GDP share -  $\omega_T = 2.97\%$  - to the population share -  $\omega'_T = 10.76\%$ . The results of the new simulation are presented in Table 6. Basically the results are pretty much the same as previously. Furthermore, the welfare cost of joining the Eurozone drops only slightly from  $\lambda = 0.068\%$  to  $\lambda' = 0.066\%$  of the lifetime consumption, or differently from  $\lambda_0 = 6.17\%$  to  $\lambda'_0 = 5.99\%$  of the period zero consumption. This only slightly lessens the macroeconomic consequences of losing autonomous monetary policy after the Eurozone accession.

Table 6: The weight of Poland and variability of the Polish business cycle

Variables	<i>Autonomous</i>	<i>Common</i>	<i>Common</i>
	<i>Monetary Policy</i>	<i>Monetary Policy</i> ( $\omega_T = 2.97\%$ )	<i>Monetary Policy</i> ( $\omega'_T = 10.76\%$ )
GDP	0.0276	0.0327	0.0325
Inflation	0.0288	0.0098	0.0097
Interest Rate	0.0261	0.0300	0.0299
Consumption	0.0106	0.0208	0.0206
Labor	0.0119	0.0134	0.01335

## 5 Conclusion

The Eurozone accession for Poland might be costly, since it means losing monetary policy as a tool to smooth out the business cycles. We build a two country dynamic stochastic general equilibrium model to quantify the effect of the loss of the autonomous monetary policy. In our model there are nominal rigidities in the form of sticky prices, which makes monetary policy not neutral in the short run. There are numerous effects associated with the Eurozone accession. This study focuses on the effects of losing autonomous monetary policy on the business cycle behaviour of the main macroeconomic variables and welfare. It is not a complete study of all the cost and benefits of the Euro adoption.

Since the evidence that higher fluctuations cause lower economic growth is scarce, we assumed that changes in the business cycle behaviour will not have any long run effect on the Polish economy. We focused on the change in the business cycle behaviour of the main macroeconomic variables. We found that after Poland joins the Eurozone, the volatility of GDP increases - the standard deviation of GDP goes up from 2.62% to 2.72%, and the volatility of inflation decreases - the standard deviation of inflation goes down from 2.41% to 0.96%. We additionally present how different factors involved with the Eurozone accession affect these outcomes.

We also calculated the impact of this change on the consumers welfare and we found that the regime change has the same effect as the decline of consumption in the period after the Eurozone accession by 6.17% (or a decline of lifetime consumption by 0.068%). Moreover, we checked how the weight of the Polish economy in the decisions of the ECB affects the results and we found that the increase in weight from 2.93% (which is equal to the GDP share) to 10.76% (which is equal to the population share) lessens the cost of joining the Eurozone, but the difference is tiny.

Furthermore, our analysis suggest that it is not enough to look at GDP and inflation to judge the impact of the autonomous monetary policy loss on the economy, because the changes in business cycle behaviour of consumption, labor and the interest rate might be greater and more important, from the welfare perspective.

We want to stress that in our model wages are flexible, which, most likely, means that our result underestimates the cost of joining the Eurozone. Another restriction that might affect the results is the existence of the representative agent - relaxing this restriction could increase the cost of business cycle fluctuations and might affect the costs of the Eurozone accession. Additionally, the introduction of unemployment into the concept of equilibrium (like e.g. in the search-matching framework) might also positively affect the costs of business fluctuations. On the other hand, the costs may be lower, if the accession to the Eurozone increases correlations of shocks or decreases their volatilities. We leave these extensions for future research.

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## Appendix

### A The Log-linearised Model

Denote the log deviation of a variable from its deterministic steady state as  $\hat{x}_t = \log\left(\frac{x_t}{\bar{x}}\right)$ . To derive the first order approximation we use the following method

$$x_t = \bar{x}x_t\bar{x}^{-1} = \bar{x}e^{(\log x_t - \log \bar{x})} = \bar{x}e^{\log \frac{x_t}{\bar{x}}} = \bar{x}e^{\hat{x}_t} \approx \bar{x}(1 + \hat{x}_t)$$

#### A.1 Households

**Capital accumulation.** From (6) we obtain

$$\hat{k}_{t+1} = (1 - \delta)\hat{k}_t + \delta\hat{x}_t, \quad (\text{A.1})$$

$$\hat{k}_{t+1}^* = (1 - \delta^*)\hat{k}_t^* + \delta^*\hat{x}_t^*. \quad (\text{A.2})$$

**Capital supply.** From (7) we obtain

$$\hat{k}_t = \hat{u}_t + \hat{k}_t, \quad (\text{A.3})$$

$$\hat{k}_t^* = \hat{u}_t^* + \hat{k}_t^*. \quad (\text{A.4})$$

**Tobin's Q.** From (11) we obtain

$$\hat{x}_t = \frac{\iota}{1 + \beta}\hat{Q}_t + \frac{\beta}{1 + \beta}E_t\hat{x}_{t+1} + \frac{1}{1 + \beta}\hat{x}_{t-1}, \quad (\text{A.5})$$

$$\hat{x}_t^* = \frac{\iota^*}{1 + \beta^*}\hat{Q}_t^* + \frac{\beta^*}{1 + \beta^*}E_t\hat{x}_{t+1}^* + \frac{1}{1 + \beta^*}\hat{x}_{t-1}^*. \quad (\text{A.6})$$

**Euler equation.** From (12) we obtain

$$\sigma E_t(\hat{c}_{t+1} - \hat{c}_t) = E_t\left[\hat{r}_{t+1} - \hat{Q}_t - \beta(1 - \delta)\left(\hat{r}_{t+1} - \hat{Q}_{t+1}\right)\right], \quad (\text{A.7})$$

$$\sigma^* E_t(\hat{c}_{t+1}^* - \hat{c}_t^*) = E_t\left[\hat{r}_{t+1}^* - \hat{Q}_t^* - \beta^*(1 - \delta^*)\left(\hat{r}_{t+1}^* - \hat{Q}_{t+1}^*\right)\right]. \quad (\text{A.8})$$

**Capital utilization.** From (13) we obtain

$$\hat{r}_t = \frac{\Psi''(1)}{\beta^{-1} - (1 - \delta)}\hat{u}_t, \quad (\text{A.9})$$

$$\hat{r}_t^* = \frac{\Psi''(1)^*}{(\beta^*)^{-1} - (1 - \delta^*)}\hat{u}_t^*. \quad (\text{A.10})$$

**No arbitrage, capital and nominal assets.** From (14) we obtain

$$\sigma E_t(\hat{c}_{t+1} - \hat{c}_t) = E_t\left(\hat{R}_t - \hat{\pi}_{t+1}\right), \quad (\text{A.11})$$

$$\sigma^* E_t(\hat{c}_{t+1}^* - \hat{c}_t^*) = E_t\left(\hat{R}_t^* - \hat{\pi}_{t+1}^*\right). \quad (\text{A.12})$$

**No arbitrage, home and international assets.** From (15) we obtain

$$\hat{R}_t - \hat{R}_t^* = E_t [(\hat{q}_{t+1} - \hat{q}_t) + (\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*)] + \hat{\kappa}_t. \quad (\text{A.13})$$

**Risk premium.** From (4) we obtain

$$\hat{\kappa}_t = \chi \frac{\bar{d}}{\bar{Y}} (\hat{d}_t - \hat{Y}_t) + \epsilon_{\kappa,t}. \quad (\text{A.14})$$

**Labor market.** From (9) and (10) we obtain

$$\sigma \hat{c}_t + \gamma \hat{l}_t + \hat{\zeta}_t = \hat{w}_t, \quad (\text{A.15})$$

$$\sigma^* \hat{c}_t^* + \gamma^* \hat{l}_t^* + \hat{\zeta}_t^* = \hat{w}_t^*. \quad (\text{A.16})$$

## A.2 Producers

Denote  $p_{H,t} = \frac{P_{H,t}}{P_t}$ ,  $p_{F,t} = \frac{P_{F,t}}{P_t}$ ,  $p_{H,t} = \frac{P_{H,t}^*}{P_t^*}$ ,  $p_{F,t}^* = \frac{P_{F,t}^*}{P_t^*}$ , and  $d_t = \frac{e_t D_{H,t+1}^*}{P_t}$ .

**Demand for homogeneous intermediate goods.** From (19) and (20) we obtain

$$\hat{Y}_{H,t} = -\frac{1+\mu}{\mu} \hat{p}_{H,t} + \hat{Y}_t, \quad (\text{A.17})$$

$$\hat{Y}_{F,t} = -\frac{1+\mu}{\mu} \hat{p}_{F,t} + \hat{Y}_t, \quad (\text{A.18})$$

$$\hat{Y}_{H,t}^* = -\frac{1+\mu^*}{\mu^*} \hat{p}_{H,t}^* + \hat{Y}_t^*, \quad (\text{A.19})$$

$$\hat{Y}_{F,t}^* = -\frac{1+\mu^*}{\mu^*} \hat{p}_{F,t}^* + \hat{Y}_t^*. \quad (\text{A.20})$$

**The inflation of intermediate goods prices.** From the definition of relative prices and inflation we obtain.

$$\hat{\pi}_{H,t} = \hat{\pi}_t + \hat{p}_{H,t} - \hat{p}_{H,t-1}, \quad (\text{A.21})$$

$$\hat{\pi}_{F,t} = \hat{\pi}_t + \hat{p}_{F,t} - \hat{p}_{F,t-1}, \quad (\text{A.22})$$

$$\hat{\pi}_{F,t}^* = \hat{\pi}_t^* + \hat{p}_{F,t}^* - \hat{p}_{F,t-1}^*, \quad (\text{A.23})$$

$$\hat{\pi}_{H,t}^* = \hat{\pi}_t^* + \hat{p}_{H,t}^* - \hat{p}_{H,t-1}^*. \quad (\text{A.24})$$

**Final goods producers.** From (21) we obtain

$$\hat{\pi}_t = (1-\eta) (\bar{p}_F)^{\frac{-1}{\mu}} (\hat{\pi}_{F,t} + \hat{p}_{F,t-1}) + \eta (\bar{p}_H)^{\frac{-1}{\mu}} (\hat{\pi}_{H,t} + \hat{p}_{H,t-1}), \quad (\text{A.25})$$

$$\hat{\pi}_t^* = \eta^* (\bar{p}_F^*)^{\frac{-1}{\mu}} (\hat{\pi}_{F,t}^* + \hat{p}_{F,t-1}^*) + (1-\eta^*) (\bar{p}_H^*)^{\frac{-1}{\mu}} (\hat{\pi}_{H,t}^* + \hat{p}_{H,t-1}^*). \quad (\text{A.26})$$

**Marginal costs of heterogeneous intermediate goods.** From (31) we obtain

$$\hat{m}c_t = \alpha \hat{r}_t + (1-\alpha) (\hat{w}_t - \hat{z}_t), \quad (\text{A.27})$$

$$\hat{m}c_t^* = \alpha^* \hat{r}_t^* + (1-\alpha^*) (\hat{w}_t^* - \hat{z}_t^*). \quad (\text{A.28})$$

**Optimal production factors employment.** From (32) we obtain

$$\hat{r}_t - \hat{w}_t = \hat{l}_t - \hat{k}_t, \quad (\text{A.29})$$

$$\hat{r}_t^* - \hat{w}_t^* = \hat{l}_t^* - \hat{k}_t^*. \quad (\text{A.30})$$

**Prices of heterogeneous intermediate goods.** From (35) and (36) we get home goods prices:

$$\begin{aligned} \hat{p}_{H,t} = & \frac{\theta_H}{1 + \beta\theta_H^2} (\hat{p}_{H,t-1} - \hat{\pi}_t) + \frac{\beta\theta_H}{1 + \beta\theta_H^2} E_t (\hat{p}_{H,t+1} + \hat{\pi}_{t+1}) \\ & + \frac{(1 - \theta_H)(1 - \beta\theta_H)}{1 + \beta\theta_H^2} \hat{m}c_t, \end{aligned} \quad (\text{A.31})$$

$$\begin{aligned} \hat{p}_{H,t}^* = & \frac{\theta_H^*}{1 + \beta(\theta_H^*)^2} (\hat{p}_{H,t-1}^* - \hat{\pi}_t^*) + \frac{\beta\theta_H^*}{1 + \beta(\theta_H^*)^2} E_t (\hat{p}_{H,t+1}^* + \hat{\pi}_{t+1}^*) \\ & + \frac{(1 - \theta_H^*)(1 - \beta\theta_H^*)}{1 + \beta\theta_H^2} (\hat{m}c_t - q_t), \end{aligned} \quad (\text{A.32})$$

and foreign goods prices

$$\begin{aligned} \hat{p}_{F,t}^* = & \frac{\theta_F^*}{1 + \beta^*(\theta_F^*)^2} (\hat{p}_{F,t-1}^* - \hat{\pi}_t^*) + \frac{\beta^*\theta_F^*}{1 + \beta^*(\theta_F^*)^2} E_t (\hat{p}_{F,t+1}^* + \hat{\pi}_{t+1}^*) \\ & + \frac{(1 - \theta_F^*)(1 - \beta^*\theta_F^*)}{1 + \beta^*(\theta_F^*)^2} \hat{m}c_t^*, \end{aligned} \quad (\text{A.33})$$

$$\begin{aligned} \hat{p}_{F,t} = & \frac{\theta_F}{1 + \beta^*(\theta_F)^2} (\hat{p}_{F,t-1} - \hat{\pi}_t) + \frac{\beta^*\theta_F}{1 + \beta^*(\theta_F)^2} E_t (\hat{p}_{F,t+1} + \hat{\pi}_{t+1}) \\ & + \frac{(1 - \theta_F)(1 - \beta^*\theta_F)}{1 + \beta^*(\theta_F)^2} (\hat{m}c_t^* + \hat{q}_t). \end{aligned} \quad (\text{A.34})$$

**Aggregate Production function.** From (28) and (22) we obtain

$$\begin{aligned} \alpha \hat{k}_t + (1 - \alpha) (\hat{z}_t + \hat{l}_t) = & \frac{\bar{Y}_H}{A \bar{k}^\alpha (\bar{z}\bar{l})^{1-\alpha}} \hat{Y}_{H,t} + \\ & + \left( 1 - \frac{\bar{Y}_H}{A \bar{k}^\alpha (\bar{z}\bar{l})^{1-\alpha}} \right) \hat{Y}_{H,t}^*, \end{aligned} \quad (\text{A.35})$$

$$\begin{aligned} \alpha^* \hat{k}_t^* + (1 - \alpha^*) (\hat{z}_t^* + \hat{l}_t^*) = & \frac{\bar{Y}_F^*}{A^* (\bar{k}^*)^\alpha (\bar{z}^*\bar{l}^*)^{1-\alpha}} \hat{Y}_{F,t} + \\ & + \left( 1 - \frac{\bar{Y}_F^*}{A^* (\bar{k}^*)^\alpha (\bar{z}^*\bar{l}^*)^{1-\alpha}} \right) \hat{Y}_{F,t}^*. \end{aligned} \quad (\text{A.36})$$

### A.3 Government

**Government expenditures.** From (38) we obtain

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \hat{\varepsilon}_{G,t}, \quad (\text{A.37})$$

$$\hat{G}_t^* = \rho_G^* \hat{G}_{t-1}^* + \hat{\varepsilon}_{G,t}^*. \quad (\text{A.38})$$

#### A.4 Central Bank

**Taylor rule.** From (39) we obtain

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1 - \gamma_R) \left( \gamma_\pi \hat{\pi}_t + \gamma_y \hat{Y}_t + \gamma_{dy} d\hat{Y}_t + \gamma_{d\pi} d\hat{\pi}_t \right) + \varphi_t, \quad (\text{A.39})$$

$$\hat{R}_t^* = \gamma_R^* \hat{R}_{t-1}^* + (1 - \gamma_R^*) \left( \gamma_\pi^* \hat{\pi}_t^* + \gamma_y^* \hat{Y}_t^* + \gamma_{dy}^* d\hat{Y}_t^* + \gamma_{d\pi}^* d\hat{\pi}_t^* \right) + \varphi_t^*. \quad (\text{A.40})$$

#### A.5 Closing the Model

**Market clearing.** From (40) we obtain

$$\frac{\bar{C}}{\bar{Y}} \hat{C}_t + \frac{\bar{X}}{\bar{Y}} \hat{X}_t + \frac{\bar{G}}{\bar{Y}} \hat{G}_t + \frac{r\bar{k}}{\bar{Y}} \hat{u}_t = \hat{Y}_t, \quad (\text{A.41})$$

$$\left( \frac{\bar{C}}{\bar{Y}} \right)^* \hat{C}_t^* + \left( \frac{\bar{X}}{\bar{Y}} \right)^* \hat{X}_t^* + \left( \frac{\bar{G}}{\bar{Y}} \right)^* \hat{G}_t^* + \left( \frac{r\bar{k}}{\bar{Y}} \right)^* \hat{u}_t^* = \hat{Y}_t^*. \quad (\text{A.42})$$

**Balance of Payments.** From (45) we obtain

$$\begin{aligned} & \left( \frac{\bar{q}\bar{p}_H^* \frac{1-\omega}{\bar{Y}} \bar{Y}_H^*}{\bar{Y}} \right) (\hat{q}_t + \hat{p}_{H,t}^* + \hat{Y}_{H,t}^*) + \left( \frac{\bar{d}}{\bar{Y}} \right) \frac{\beta^*}{\bar{\pi}^*} (\hat{d}_t - \hat{R}_t^* - \hat{\kappa}_t) = \\ & = \left( \frac{\bar{p}_F \bar{Y}_F}{\bar{Y}} \right) (\hat{p}_{F,t} + \hat{Y}_{F,t}) + \left( \frac{\bar{d}}{\bar{Y}} \right) \frac{1}{\bar{\pi}^*} (\hat{d}_{t-1} + \hat{q}_t - \hat{q}_{t-1} - \hat{\pi}_t^*). \end{aligned} \quad (\text{A.43})$$

**GDP.** From (46) we obtain

$$\begin{aligned} \hat{Y}_t = & \left( \frac{\bar{Y}}{\bar{Y}} \right) \hat{Y}_t + \left( \frac{\bar{q}\bar{p}_H^* \frac{1-\omega}{\bar{Y}} \bar{Y}_H^*}{\bar{Y}} \right) (\hat{p}_H^* + \hat{Y}_H^* + \hat{q}_t) - \\ & - \left( \frac{\bar{p}_F \bar{Y}_F}{\bar{Y}} \right) (\hat{p}_F + \hat{Y}_F), \end{aligned} \quad (\text{A.44})$$

$$\begin{aligned} \hat{Y}_t^* = & \left( \frac{\bar{Y}^*}{\bar{Y}^*} \right) \hat{Y}_t^* + \left( \frac{\bar{p}_F \frac{\omega}{1-\omega} \bar{Y}_F}{\bar{q}\bar{Y}^*} \right) (\hat{p}_F + \hat{Y}_F - \hat{q}_t) - \\ & - \frac{\bar{p}_H^* \bar{Y}_H^*}{\bar{Y}^*} (\hat{p}_H^* + \hat{Y}_H^*). \end{aligned} \quad (\text{A.45})$$

## B Welfare Cost Computation

Here we describe the details of computing  $\lambda$ . First notice that it is much more convenient to solve

$$\frac{W^{OUT}(\lambda)}{\bar{c}^{1-\sigma}} = \frac{W^{IN}}{\bar{c}^{1-\sigma}}, \quad (\text{B.1})$$

rather than (53). Next we split total utility into utility from consumption and disutility from work. Then for any regime  $\Theta \in \{IN, OUT\}$

$$\frac{W^\Theta(\lambda)}{\bar{c}^{1-\sigma}} = \frac{W^{C,\Theta}(\lambda)}{\bar{c}^{1-\sigma}} - \frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}}, \quad (\text{B.2})$$

where

$$\frac{W^{C,\Theta}(\lambda)}{\bar{c}^{1-\sigma}} = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{((1-\lambda)c_t^\Theta)^{1-\sigma}}{1-\sigma} \right], \quad (\text{B.3})$$

$$\frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}} = E_0 \left[ \sum_{t=0}^{\infty} \beta^t \psi \zeta_t \frac{(\hat{l}_t^\Theta)^{1+\gamma}}{1+\gamma} \right]. \quad (\text{B.4})$$

It is convenient to denote  $W^{C,\Theta} \equiv W^{C,\Theta}(1)$ , than  $\frac{W^{C,\Theta}(\lambda)}{\bar{c}^{1-\sigma}} = (1-\lambda)^{1-\sigma} \frac{W^{C,\Theta}}{\bar{c}^{1-\sigma}}$ . To find  $\frac{W^{C,\Theta}}{\bar{c}^{1-\sigma}}$  and  $\frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}}$  first notice that (9) and (10) imply that in the deterministic steady state

$$\varrho \bar{c}^{1-\sigma} = \psi \bar{l}^{1+\gamma}. \quad (\text{B.5})$$

After straightforward computations of the equilibrium conditions in the deterministic steady state we get that

$$\varrho = \frac{(1-\alpha)[1-\beta(1-\delta)] \left(\frac{\bar{x}}{\bar{y}}\right)}{\alpha \delta \beta \left(\frac{\bar{c}}{\bar{y}}\right)} \quad (\text{B.6})$$

which we calibrated to match the deterministic steady state relationships and we found that  $\varrho = 1.09$ . Next we used (B.5) to find the following

$$\frac{W^C}{\bar{c}^{1-\sigma}} = \sum_{t=0}^{\infty} \beta^t \frac{(\exp(\hat{c}_t))^{1-\sigma}}{1-\sigma}, \quad (\text{B.7})$$

$$\frac{W_L}{\bar{c}^{1-\sigma}} = \sum_{t=0}^{\infty} \beta^t \varrho \zeta_t \frac{(\exp(\hat{l}_t))^{1+\gamma}}{1+\gamma}. \quad (\text{B.8})$$

Notice that with this formula it is not necessary to find the deterministic steady state values of  $\bar{c}$  or  $\bar{l}$ .

To find the welfare cost expressed in period zero consumption  $\lambda_0$  we use analogous method. The only difference is that (B.1) is replaced with

$$\frac{W^{OUT}(\lambda_0)}{\bar{c}^{1-\sigma}} = \frac{W^{IN}}{\bar{c}^{1-\sigma}}, \quad (\text{B.9})$$

where

$$\frac{W^\Theta(\lambda_0)}{\bar{c}^{1-\sigma}} = E_0 \left[ \frac{((1-\lambda_0)c_0^\Theta)^{1-\sigma}}{1-\sigma} + \sum_{t=1}^{\infty} \beta^t \frac{(c_t^\Theta)^{1-\sigma}}{1-\sigma} \right] - \frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}} \quad (\text{B.10})$$

$$= (1-\lambda_0)(1-\beta) \frac{W^{C,\Theta}}{\bar{c}^{1-\sigma}} + \beta \frac{W^{C,\Theta}}{\bar{c}^{1-\sigma}} - \frac{W^{L,\Theta}}{\bar{c}^{1-\sigma}}. \quad (\text{B.11})$$